Problem 1
Implicit Function Rule

There are two sectors in the economy, A and B. They have their own capital – K_A and K_B – which are given and constant. They share a pool of labor L, however, so that:

$$L = L_A + L_B \tag{1}$$

Output is produced with the following production functions:

$$Y_A = K_A^{\alpha} L_A^{1-\alpha} \tag{2}$$

and

$$Y_B = K_B^{\beta} L_B^{1-\beta} \tag{3}$$

Here, we constrain $0 < \beta < \alpha < 1$.

Assume wages are equal to marginal products of labor (perfect competition). Assume workers are mobile, so they move to the sector where wages are higher. In equilibrium, we know that

$$w_A = w_B \tag{4}$$

Assume the economy is in equilibrium and then industry A recieves a gift of capital, so that $dK_A > 0$. Find:

$$\frac{\partial L_A}{\partial K_A} \tag{5}$$

using the Implicit Function Rule. That is, how much labor (per unit of capital) moves to the A-Sector after capital is increased there?