Economic Growth and Development: The Growth Book

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Contents

Chapter 1

Growth and Development: The Principal Issues

1.1 Introduction

The Industrial Revolution ushered in an era of wide disparity of real per capita income across nations. This disparity is illustrated in Figure 1.1, which shows the progress of countries over the last 400 years. Although the data is quite imprecise before 1850, this figure illustrates three important points:

- The phenomenon of great differences in living standards is fairly recent in history.
- The disparity today is staggering: the ratio of the best to the worst is about 32.
- For these differences to have developed, there must have been a time of tremendous *growth* differences.

The clear implication of Figure 1.1 is that the only way for a catch-up to occur is for less-developed countries (LDC's) today to grow considerably *faster* than the industrial countries. While some progress is being made, notably in Southeast Asia and Eastern Europe, many nations around the

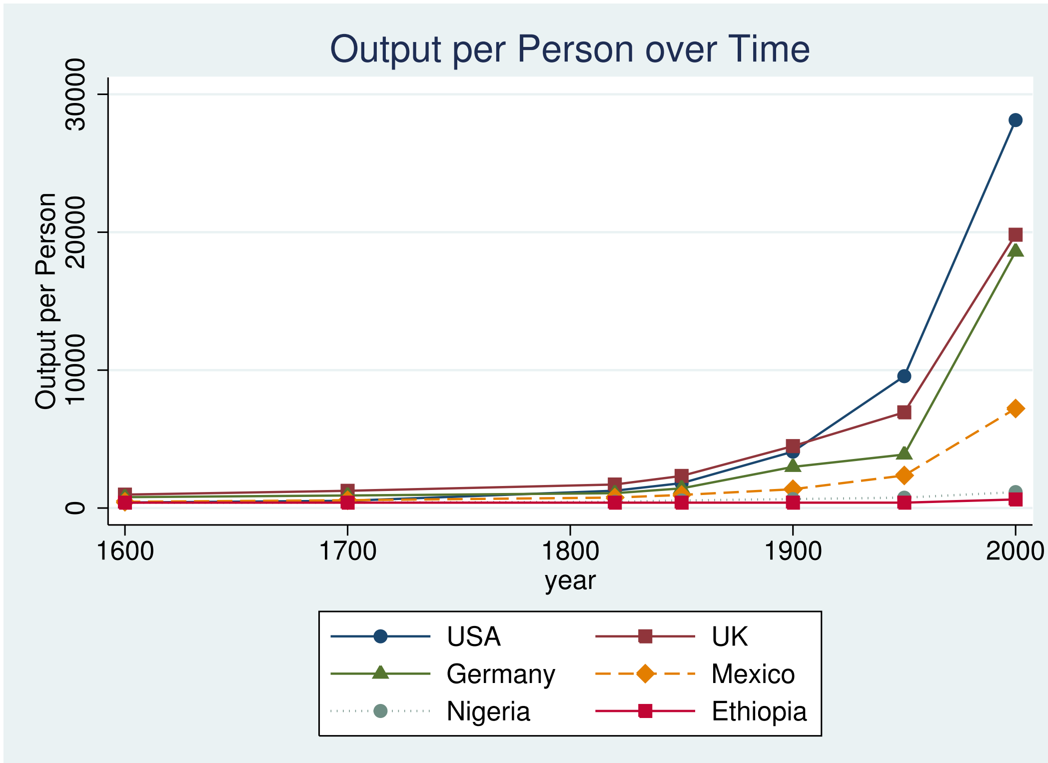

Figure 1.1: Output per Person in History

world show virtually no tendency to raise their growth rates above those of the industrial West, and many more are stagnating, falling increasingly far behind world leaders. There may be no more valuable exercise than trying to understand why certain nations were able to raise their growth rates above normal to achieve high real, per capita income, while others have not been able to do so for centuries.

As a first step in the process of understanding, it is important to be able to measure how well different countries are doing. We take up this difficult task in **Part 2** of the course (see Reading List), where we look at methods to adjust GDP in different years and different countries to make them comparable.

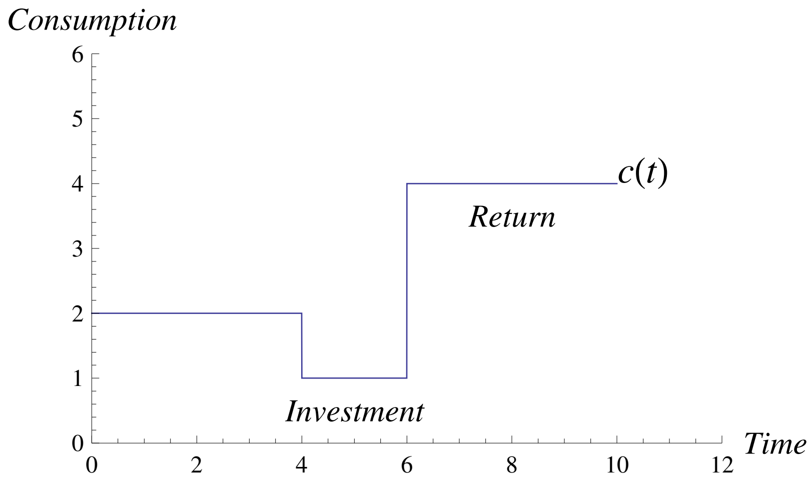

Figure 1.2: Investment and Return

1.2 The Sources of Economic Growth

Two processes lie at the heart of growth: *accumulation* and *innovation*.

Consider Figure 1.2, which shows the consumption stream of a primitive fisherman. If she takes half a day off for a week to build a net – which involves a current sacrifice of consumption — she will have the means to consume more fish forever. Growth and development usually require the sacrifice of current consumption and leisure to accumulate capital. People can accumulate both *physical capital* and *human capital* (education or skill). Both of these require a current sacrifice that will raise output permanently later. In the early stages of thinking about growth and development, the accumulation (or saving) process was thought to be of primary importance. We look at basic accumulation in **Parts 4 and** 5.

The return to capital accumulation (or the productivity of capital) depends on the state of knowledge or technology. The accumulation of a unit of capital today yields far more output than investment in the last century because of the vast rise in our knowledge. Innovation refers to the process by which practical, technical knowledge increases. Although advances in basic science are sometimes necessary for improvements in industrial technology, that is not always the case. In many historical periods (the Roman Empire and China, for example) useful technology lagged far behind what was possible given the state of scientific knowledge. We begin looking at technology in **Parts 6 and** 7, but it is an issue that comes up repeatedly in different contexts.

Innovation may also take place via imitation. For many nations, it is far easier to adapt existing technology rather than create new technology. This is especially true where intellectual property rights are weak. One might also consider specialization itself to be a type of innovation. It certainly is the source of higher real income per person, and thus of growth.

1.3 Factors That Contribute to the Basic Processes

Countries are not all alike. Some of the differences involve physical resources and some have to do with culture, both of which may be considered to be exogenous; that is, given from the beginning and difficult to influence by policy-makers. Many variables that we observe, however, can be changed by the government, at least in the long run. This section looks at three broad characteristics of nations that are often singled out as exerting an influence on economic development. In each case, they influence the fundamental processes noted above.

1.3.1 Population and Scale

For centuries, thinkers have been concerned about population's effect on economic growth. Before Malthus (around 1800), population was considered a positive force for growth. Now, in many circles the reverse is true: population is thought of as an impediment to growth.

There are essentially two reasons that a large population can raise living standards and growth rates: *scale economies* and *idea generation.* We consider these ideas beginning in **Part 3** and continuing in **Part 10**.

Economies of scale — meaning a rise in per capita output as people increase in number —arise from the division of labor that accompanies the specialization mentioned above. When people concentrate on one task, they get better at it and save time since they need not keep switching between tasks. According to Adam Smith: "The division of labor is limited by the extent of the market." The market can be extended in two ways: by the natural growth of population and by the reduction of transport costs. It is not clear if we want to think of specialization gains as involving new technologies, or simply utilizing existing ones better. In either case, a larger population will make everyone better off through specialization.

Ideas come from people, perhaps interacting with others. The second way that greater numbers raise per capita income is through their ability to create more and better ideas for production and distribution. In this view, both kinds of technology are thought to increase faster when population is greater, simply because ideas come from people, so the more there are, the greater the chance of finding a new invention.

The above processes work in the long term. In the short run, however, it is possible that an increase in population would reduce per capita income. The reason is that all other factors, like land and machinery, are fixed in the short run so that more workers are likely to encounter diminishing marginal productivity of their effort. Think about putting more and more laborers to work on a farm of fixed acreage: although output will rise at first, sooner or later the productivity of new workers would be become so low that production per person would fall. Today, this appears to be a significant problem in traditional societies in Sub-Saharan Africa and parts of south Asia.

1.3.2 Openness and Geography

Some countries are very open to the outside world in terms of trade in goods and the exchange of ideas. Others, like Cuba and North Korea, are almost totally closed to imports of commodities and technologies. The static gains from trade are well-known; nations that refuse to trade can be expected to have lower levels of per capita income compared to comparable countries that are open, since they do not allow themselves to take advantage of the benefits of international specialization and economies of scale.

The effect of openness on growth and development is more controversial. Indeed, many policies that serve to restrict commerce are justified by appeal to their effects in enhancing industrialization and development. The idea, which was very popular in the period directly following WWII, is that industry is valuable because it creates positive external effects on the rest of the economy. So even if the industry makes losses, its benefit to society may be so great that the enterprise is worthwhile, and worthy of a subsidy. External effects, both positive and negative, are very difficult to measure, however, so it was never clear which industries, if any, should be protected. Indeed, one could argue that the opposite policy was better: the country should remain open to new ideas and goods to generate faster growth through technology transfer and human capital development. Globalization is the latest form of the debate about openness. There are several fascinating aspects to this debate, one of which we discuss in **Part 8**.

Apart from openness, there is the question of geography. Do the physical and spatial characteristics of a country make a difference for development? Currently, there has been a renewal of the interesting debate concerning the tropics. No industrial country is located within the tropics. Is this because of the climate and geographical features of these countries, or does it stem from the colonial heritage? We look into these issues in **Part** 13.

1.3.3 Government

One of the most controversial areas of development economics concerns the role of the government and politics in general. Is the government a positive force for growth, performing collective functions that would not be performed by the market? Or is it a negative force, one that interferes in the growth process by imposing inefficient redistributive policies on the market?

The early literature focused on the government's ability to plan and mobilize large-scale resources for development. Moreover, it is reasonable to suppose that a government is necessary for the provision of several key public goods, like education, public health, and transportation.

Focusing on a large, involved government led to the policy of restricting imports to encourage industry, as noted above. People conceived of the government as a powerful, yet benevolent, force acting to counter the ill effects of a market system that did not function to maximize general welfare. More recently, however, the literature has concentrated on the failures of the government to carry out such a role. Government corruption, incomplete property rights, exploitation, and political decision-making can all hamper growth. The government's proper role may small: to enforce property rights and contracts, and provide essential public services so all can realize their full potential.

When looked at in this way, development is seen to depend crucially on the nation's institutions. If the proper institutional environment is in place, development will proceed almost automatically. **Part 9** deals with these issues.

In **Part 12** we discuss the particular problem of sustainability and growth.

1.4 Conclusion

Economic development depends on the interplay of population, technology, and institutions. Not only do these exert a powerful influence on growth and structural change, they are interrelated in complex ways. Coming to a basic understanding of these relationships and their effect on growth is the main purpose of this course. Policy for development can only succeed to the extent that it is based on sound fundamental theory informed by history.

Chapter 2

International Comparison of Income across Time and Space

2.1 Introduction

How do we know if a country is better off than others? More precisely, how do we measure how much better one country is than another? If we are to propose useful theories of growth, it is necessary that we also have means of validating those theories. We can only do that if we can measure relative performance in both dimensions: across time and across space.

The problem is difficult because the people of different nations consume different items and use different currencies. In this chapter, we show how adjustments to the data are made to improve the comparison. It should always be kept in mind that these adjustments are imprecise and that we cannot find exact measures of national product that are comparable between countries and between time periods.

2.2 The Exchange Rate

We begin by assuming that we know the nominal GDP of a country; that is, the total value of final output, expressed in the home currency. If there are m different goods:

$$Y = p_1 q_1 + p_2 q_2 + p_3 q_3 + \ldots \; = \sum_{i=1}^{m} p_i q_i = \mathbf{p'}_i \mathbf{q_i} \tag{2.1}$$

where the bolded symbols are vectors of prices and quantities.[1] For every country, $j = 1,,2,\ldots n$, we have such a measure. It will be in yen, euros, dollars, pounds, etc. We assume that each of the n countries has m different goods.

The measures in (2.1) are in different currencies so we must express them all in terms of a common currency in order to compare them. Perhaps the most obvious thing to do is to use the US dollar exchange rate to express all of them in terms of US money:

$$\tilde{Y}_j = \frac{Y_j}{E_j} \tag{2.2}$$

Here, E_j is the exchange rate in, for example, € /\$ or £/\$; that is, E_j is the *j-currency price of one US dollar*, so that $\tilde{Y}_j$ is a measure of Nominal GDP in terms of dollars. Think about the price of pizza in England: $18.95 \frac{\pounds}{Pizza}$. It is a ratio, too. The price of a dollar in the UK is analogous.

This measure is not satisfactory because the exchange rate is not a good measure of comparable goods value. What I mean by this is that if you took \$100 you could buy a lot more in India (after conversion to Rupees at the current exchange rate) than you could in the US. But in Switzerland you would not be able to get nearly as much as in the US. Exchange rates do not do a very good job in making currencies equal *in terms of the goods they will purchase*. Yet, to compare living standards between different nations, that is precisely what we need: a conversion factor based on goods.

[1] In this expression, $\mathbf{p'}$ is a row vector , while $\mathbf{q}$ is a column vector.

2.3 A Basic Measure: The Big Mac Factor

Because of these problems, we want to invent a new “exchange rate”, a synthetic one, that we can use to convert one country’s GDP into units that make it directly comparable to another’s GDP. One very simple way to do this is to assert that one good, a Big Mac, for example, “should” cost the same in every country, since it is a fairly standard product. If so, then the ratio of the two hamburger prices implicitly defines an “exchange rate”. In the case of the United States and Great Britain we have:

$$F^M = \frac{p^{\pounds}_{Mac}}{p^{\$}_{Mac}} \tag{2.3}$$

A good way to think of F^M is as follows: if the market exchange rate E were equal to F^M, then a Big Mac would cost the same in the US and the UK (that is, it would take the same number of dollars to buy a Big Mac, whether you did so in the US, or bought £’s first then bought a Big Mac in London). A recent search of the web came up with the following data for Big Mac prices in the UK and the US: $p^{\pounds}_{Mac} = \pounds 2.99$ and $p^{\$}_{Mac} = \$5.33$. This means that $F = 0.5610 \frac{\pounds}{\$}$.

Note that F^M has the dimension £/\$, just like E, and is, in some sense, an ideal exchange rate. Since the Big Mac can be found in virtually every country on earth (an exaggeration) we could use F^M in place of E in (2.2) and get a better measure of dollar GDP for each country. These can then be compared across countries. In fact, *The Economist* magazine began doing this decades ago.

The technical term for a factor like F^M is a “Purchasing-Power-Parity Exchange Rate” or PPP-ER for short. For F^M to be a “perfect” factor requires that the prices of all goods in the country stand in the same relation to their US counterpart goods as do Big Macs. That is, if F^M were a perfect measure, if you took the ratio of *any* two items (bicycles, for example) you would get the same number shown in (2.3). This, of course, would never happen, if for no other reason than people’s preferences vary widely over the globe, and transportation costs are significant and different.

The actual exchange rate in early 2018 was $E = 0.73692\frac{£}{\$}$, which is *more* than the F^M found above. In this case, we say that *the dollar was overvalued* and *the pound was undervalued*. That is, since E is the price (value) of the dollar, when $E > F^M$ it means that the dollar's market (actual) value is greater than its "ideal" value. Hence, the market overvalues the dollar (and undervalues the pound).

The Big Mac rate F^M is way too simple a substitute for E. Yet it works, and conveys the objective of the whole comparison project in a relatively simple way: we are looking for factors with which we can convert a country's GDP in order to make it directly comparable to that of all other countries.

Look at the row for the year 2000 in Table 2.1. Each entry shows y_{j2000}, defined to be Country j's GDP in the year 2000 expressed in "purchasing-power" US dollars of 2000. Each entry is constructed as follows:

$$y_{j2000} = \frac{Y_{j2000}}{F^M_{j2000}} \tag{2.4}$$

where j refers to the country and 2000 refers to the year. In the row for the year 2000, there are three entries, one for the US, one for Argentina, and one for any country j. In the complete table, of course, there would be 190 columns, one for each country.

These entries are GDP of each country, expressed in "US dollars of 2000". That is, by dividing the value of each nation's GDP in 2000 – expressed in its own currency – by its particular F^M_j in (2.3) converts its GDP to its value in terms of US dollars of 2000. If we put actual numbers in for the expressions, we could compare each cell to see just how much one nation's living standard exceeded that of another in 2000.

Now we consider the additional complexity of comparing across time. Consider only the USA, shown in the first column of Table 2.1. How can we compare GDP of 1950 to that of 2000 or 2018, given that prices have increased over time, so the dollar was worth less in the later years compared to the earlier years? The easiest way to proceed is to form the following "exchange rate" between "dollars of Year t" and "dollars of 2000" In the example below, the year 2000 will be considered the *base year*, so to find

Table 2.1: Comparing Output per Capita

	Country		
Year	USA	Argentina	Country j
1950	$y_{US50} = \frac{Y_{US50}}{\Pi_{50}}$	$y_{Ar50} = \frac{Y_{Ar50}}{\theta_{Ar50}}$	$y_{j50} = \frac{Y_{j50}}{\theta_{j50}}$
...			
2000	$y_{US00} = Y_{US00}$	$y_{Ar00} = \frac{Y_{Ar00}}{F^M_{Ar00}}$	$y_{j00} = \frac{Y_{j00}}{F^M_{j00}}$
. . .			
Year t			$y_{jt} = \frac{Y_{jt}}{\theta_{jt}}$
...			
2018	$y_{US18} = \frac{Y_{US18}}{\Pi_{18}}$		$y_{US18} = \frac{Y_{US18}}{\theta_{18}}$

out how much the price of a Big Mac has changed between 1950 and 2000 we form the following ratio:

$$\Pi_{1950} = \frac{p^{\$1950}_{Mac}}{p^{\$2000}_{Mac}} = \frac{.63\,\$_{1950}}{3.81\,\$_{2000}} = .165\frac{\$_{1950}}{\$_{2000}} \tag{2.5}$$

The ratio in (2.5) is the "price" of a 2000-dollar in units of a 1950-dollar (the actual prices are made up, but the ratio reflects the US price inflation). This is an unusual concept; we are not used to thinking of exchange rates for the same money across time! The way to think of it is this: it would take only .165 dollars in 1950 (that is, 16.5¢) to buy the same goods that 1 dollar bought in 2000. A dollar went a lot farther back then. Another way of saying this is that we have had inflation of $\left(\frac{1}{\Pi} - 1\right) * 100 = 505$ percent in those 50 years!

It might be useful to assume that money changed color whenever it changed value! That is, if money were green in 1950, it might be red in 1980 and blue in 2000. Then we could talk about the price of a red dollar in terms of blue dollars. Or vice versa.

To adjust GDP in 1950, we divide it by $\Pi_{50} = .165$ to make it comparable to GDP of 2000. To take a later year, say, 2005, if $\Pi_{2005} = 1.12$ then we would scale back GDP in 2005 to 1/1.12 of its recorded value to make it comparable to that of 2000. In Column 1 of Table 2.1 we multiply the US current GDP by the appropriate Π_t to get y_{USt}, the value of US output in year t, expressed in dollars of 2000.

For this method to be useful, we must assume that prices in general in the US have increased about as much as the price of a Big Mac over that 50 year span. That may be fairly accurate (although durable goods prices have risen much less than food, and some have fallen). In any case, we have now achieved comparability between all of the entries for *Column* 1 and *Row* 2000.

Now, we want to find all of the y_{jt}: the value of per capita GDP in Country j in Year t expressed in US dollars of 2000. That is, we want to complete Table 2.1.

To do so, we take the national GDP per capita in Year t, divide it by F^M_{jt} of that year for that country, then divide it by the appropriate Π_t . Take Brazil in 1972, for example. First find the value of Brazil's per capita output in 1972 in terms of Cruzeiros of 1972 – then divide it by F^M_{Br1972} to get it in terms of US$ of 1972; finally, divide it by Π_{1972} to get it in terms of US$ of 2000.

To make Table 2.1 less cluttered, define the following factor:

$$\theta_{jt} = F^M_{jt}\Pi_t \tag{2.6}$$

where j stands for the country and t stands for the year. Then, we have:

$$y_{jt} = \frac{Y_{jt}}{\theta_{jt}} \tag{2.7}$$

The entries of Table 2.1 shows how to convert the GDP of any country in any year to make it comparable to US GDP in 2000.

2.4 The International Comparison Project

The International Comparison Project (ICP) of the United Nations is the leading group working on finding the best possible measures of comparable GDP. Basically what they do is go to each country and collect price data on a few hundred different items, divided into 3 main categories, Consumption, Investment, and Government. The ultimate goal is to find the best *average* price ratios to serve as an alternative to the Big Mac F^M.

In the process of finding the preferred measure, the Project generates several different, useful variables. Appendix A shows how they find the PPP-ER based on the US dollar, by using the price of an "average" good in each country in place of the Big Mac. Let us call this hypothetical rate F. It can be described as follows:

$$F_{jt} = \frac{Price\,of\;Ave\,Good\,in\,Country\,j\,in\,Year\,t}{Price\,of\;Ave.\,Good\,in\,US\,in\,Year\,t} \tag{2.8}$$

All constructed PPP-ER's are trying to measure the ratio displayed in (2.8). It is just like the Big Mac rate, except we have substituted "the average good" for a Big Mac.

This PPP-ER measure, although more sophisticated than our earlier measure based on Big Macs, does exactly what the simple one did: it measures "the currency-j price of 1 dollar" in the sense of defining the "ideal" currency value (or exchange rate) that would allow you to buy the same amount of goods in Country j and the US given that you had, say, \$100.

Now in Table 2.1, wherever there is an F^M, just substitute F , and you have a very good measure of comparable GDP values. For Π_t we would use a measure of average prices in the US across the different time periods (not just the Big Mac price in the US across time). And we would adjust θ to be $\theta_{jt} = F_{jt}\Pi_t$.

In constructing the F_{jt}'s, they also produce PPP-ER measures for each of the items and categories (Consumption, Investment, Government expenditure) separately. Sometimes these are very useful since different theories of growth predict differences in prices between sectors.

One of the most important sources of inter-country data based on the ICP is the **Summers-Heston-Aten Penn World Table (PWT) data set**,[2] which we discuss throughout this class. There are *five* different measures of national GDP measured in US dollars. We will be most concerned with the measure of total domestic product based on expenditure:

$$CGDP^e_{jt} = \frac{Y_{jt}}{F_{jt}} \tag{2.9}$$

where Y_{jt}, defined in Equation (2.2), is the national-currency value of GDP. By dividing by F_{jt}, which is the PPP-ER in units of (curr j/\$), we convert it to US dollars. (See Appendix 2.8 for more on the construction of the PPP-ER we call F).

The construct $CGDP^e$ allows us to compare GDP across countries for every year that they are made, but they do *not* allow us to compare across different years. Because it is so difficult to gather data on prices around the world, the ICP has constructed values of F for only seven years. These are the so-called "Benchmark Years": 1970, 1975, 1980, 1985, 1996, 2005, and 2011. Through a complicated process, they interpolate and extrapolate the F's, which allows them to find $CGDP^e$ for all years. In a similar way, they construct a variable $CGDP^o$, which treats imports and exports more directly. The difference between them is this: $CGDP^e$ is useful for analyzing the standard of living, while $CGDP^o$ is useful for analyzing productive capacity and productivity. In neither case, however, can we use these to compare across *time*. They are only comparable across countries in a given year.

To shed more light on this issue, consider what it means for $\frac{F}{E} < 1$, where F is the PPP-ER and E is the actual dollar-price of the currency. Consider the case of Mexico, which uses pesos. Using (2.8), we can express the condition as follows:

[2]This data, which is used in many research papers, is available free on the net at: https://www.rug.nl/ggdc/productivity/pwt/

$$\begin{aligned}
\frac{F_{\$}^{peso}}{E_{\$}^{peso}} &< 1 \\
\Rightarrow \frac{Ave\,P\,in\,Mexico}{Ave\,P\,in\,US} &< E_{\$}^{peso} \qquad (2.10) \\
\Rightarrow Ave\,P\,in\,Mexico &< (Ave\,P\,in\,US) * E_{\$}^{peso}
\end{aligned}$$

Thus, when $F < E$ (so that $\frac{F}{E} < 1$) the price level in the home country (Mexico) is *below* the US price level, when converted at the going exchange rate. It is cheaper to live in Mexico than the US if you begin with dollars.

The other interpretation of $F < E$ is that the home currency is *undervalued* – since the dollar is overvalued: its market price exceeds its ideal price.

The PWT data set is used in a great deal of the empirical research in economic growth. It reports both the ratio noted above $\frac{F}{E}$ and the nominal exchange rate for each country. Interestingly, it calls $\frac{F}{E}$ "the price level". The "Price Level" for Country j is therefore:

$$PL_j = \frac{F_j}{E_j} \times 100 \qquad (2.11)$$

The name "price level" is a bit misleading: given our discussion above, PL is best thought of as *the price level relative to the that of the United States.* Note that, by the definition in (2.11), $PL_{US} = 100$, since both the numerator and the denominator are 1. Moreover, the price level for any country whose F_j is the same as its E_j also has a price level of 100. In 1990 PL_{India} was only 26. The cost of living in India, by this measure, was 26 percent of its level in the US. We can, equivalently, say that the rupee was *undervalued* relative to the US dollar in 1990.

The PWT reports both PL_j and E_j, so you can multiply the two to find F_j. Again, what you end up with is a "true" measure of the (National currency)/(US dollar) exchange rate.

2.5 Comparing Across Time As Well as Space

Along with its own collected data, the ICP project uses National Account Data from the various countries as reported by the United Nations and World Bank to complete a matrix analogous to that shown in **Table** 2.1. As noted earlier, there are *five different series* for GDP reported in the PWT 9.0 data set. We have discussed two already: $CGDP^e$ and $CGDP^o$. The other three are "real" series in that they are comparable across time. These are called $RGDP^e$, $RGDP^o$, and $RGDP^{NA}$. We could, for example, compare $(RGDP^e/N)^{US}_{1965}$ with $(RGDP^e/N)^{Mexico}_{1985}$ if we wanted to see how the US living standard in 1965 compared with Mexico's living standard in 1985. (N is population.) We could do the same with the "o" and "NA" measures, but they tell us slightly different things. From now on, we concentrate on $CGDP^e$ and $RGDP^e$.

To take a numerical example, consider the numbers in **Table** 2.2 taken from the PWT v. 9.0, for which the base year is 2011. That means that in 2011, $CGDP^e = RGDP^e$. This table compares the progress of Ireland and the United States from 1990 to 2014 (the last year for which we have data). The entries are for *per capita* output.

In 2011, the base year, $CGDP^e = RGDP^e$ for both Ireland and the US, but the per capita value was higher for the US, as we would probably expect. The column labeled "L" shows the ratio $(CGDP^e/N)^{Irl}_t \,/\, (CGDP^e/N)^{US}_t$. This gives us the relative living standard in Ireland compared to the US in Year t. Note the remarkable progress made by Ireland since 1990: the standard of living rose from 49 percent of the US standard to 93 percent of the US standard.

The column labeled "f" shows the ratio $f = \left(RGDP^e_t/RGDP^e_{RefY}\right)^{US}$. This ratio shows the standard of living *in the US* in year t relative to a "reference year", which in this case is the base year of 2011 (but it could be any year). Note that US living standards have increased by about 44 percent between 1990 and 2014, during which time f rose from .73 to 1.05.[3] So, while the US has made solid progress, Ireland has done even better: if

[3]That is, 1.05/.73=.4386. The change is 1.05 - .73 = .32, which is 43.86 percent of .73.

Table 2.2: Ireland and the United States

Measures of Comparable GDP Per Capita								
	Ireland		United States		Parente/Prescott			PWT
Year	$CGDP^e$	$RGDP^e$	$CGDP^e$	$RGDP^e$	L	f	$Q = L \times f$	Z
1990	18,087	18,006	36,620	36,398	0.49	0.73	0.36	0.36
1991	18,319	18,235	36,202	35,994	0.51	0.72	0.36	0.37
1992	18,865	18,767	37,165	36,955	0.51	0.74	0.38	0.38
1993	19,792	19,698	37,934	37,722	0.52	0.76	0.39	0.39
1994	20,935	20,956	38,939	38,977	0.54	0.78	0.42	0.42
1995	23,499	23,609	39,532	39,621	0.59	0.79	0.47	0.47
1996	25,114	25,392	40,559	40,775	0.62	0.82	0.51	0.51
1997	27,903	28,266	41,841	42,283	0.67	0.85	0.56	0.57
1998	30,968	31,519	43,150	43,902	0.72	0.88	0.63	0.63
1999	33,053	33,599	44,695	45,473	0.74	0.91	0.67	0.67
2000	35,838	36,245	46,078	46,740	0.78	0.94	0.73	0.73
2001	37,402	37,627	46,290	46,731	0.81	0.94	0.76	0.75
2002	39,529	39,590	46,809	47,116	0.84	0.94	0.80	0.79
2003	39,756	39,784	47,578	47,977	0.84	0.96	0.80	0.80
2004	40,823	40,824	48,943	49,398	0.83	0.99	0.83	0.82
2005	43,096	42,547	50,783	50,512	0.85	1.01	0.86	0.85
2006	46,016	45,728	51,255	51,374	0.90	1.03	0.92	0.92
2007	49,801	49,726	51,442	51,734	0.97	1.04	1.00	1.00
2008	45,998	45,736	50,482	50,439	0.91	1.01	0.92	0.92
2009	42,859	42,888	48,591	48,840	0.88	0.98	0.86	0.86
2010	43,511	43,569	49,431	49,596	0.88	0.99	0.87	0.87
2011	45,014	45,014	49,909	49,909	0.90	1.00	0.90	0.90
2012	45,725	45,874	50,657	50,752	0.90	1.02	0.92	0.92
2013	46,166	46,575	51,005	51,317	0.91	1.03	0.93	0.93
2014	48,283	48,767	51,983	52,292	0.93	1.05	0.97	0.98

it had not, then its living standard still would be only 49 percent of that of the US.

The PWT data also includes measures for the broad categories C, I, G, as well as imports and exports, in the same way so that researchers can compare, say, investment spending in Brazil in 1980 with that of France in 1990.

There are various ways to use the PWT data. It is most useful for comparing the performance of different countries around the world. In Table 2.2 we compared Ireland and the US over time in a particular way, how Ireland did relative to the US in any single year. Another way to proceed is to examine the *distribution* of real output of all countries.

First, we look at the distribution of per capita output relative to the US in the last year for which we have data, 2014:

$$L_{j2014} = \frac{(CGDP^e/N)_{j2014}}{(CGDP^e/N)_{US2014}} \tag{2.12}$$

This is shown in **Figure** 2.1. We expect that most of the L_j values would be less than 1.0. This is borne out in **Figure** 2.1. There are only 10 countries with $CGDP^e/N$ above that of the US (most are oil exporters like Brunei and Norway, but Switzerland and Singapore are among them, too).

Now we look at the distribution of:

$$Q_{j2014} = L_{j2014} \times f_{1950} \tag{2.13}$$

where f_{1950} is the measure of relative living standards *in the US* in 2014 relative to 1950. That is, f_{1950} is $(RGDP^e_{2014}/RGDP^e_{1950})^{US}$. The variable $Q_{jt} = L_{jt}f_{ry}$ (where ry is the reference year) shows us how well Country j is doing relative to the US in the reference year, which is 1950 in this case. This distribution is shown in **Figure** 2.2. The interesting thing about this graph is that in 2014 about half the countries in the world were worse off than the United States in 1950.

Return to **Table** 2.2. The column labeled "$Q = L * f$" shows Irish output per capita in year t relative to the US base year of 2011. That is, the f here

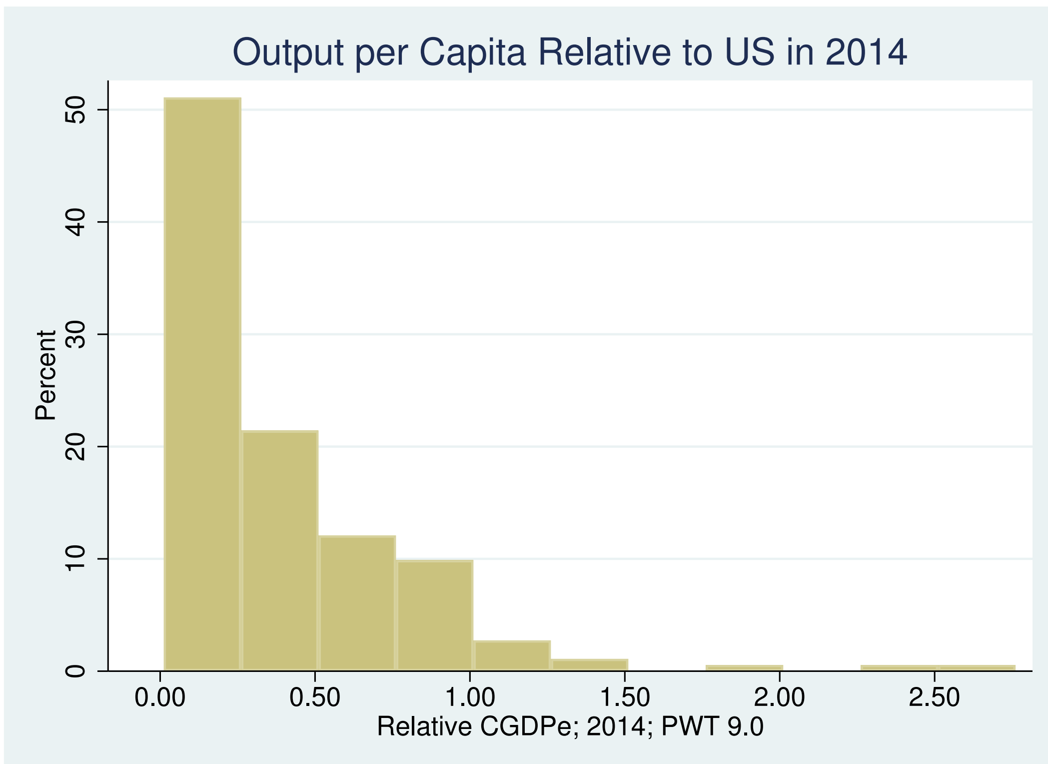

Figure 2.1: Distribution of Relative Output per capita in 2014

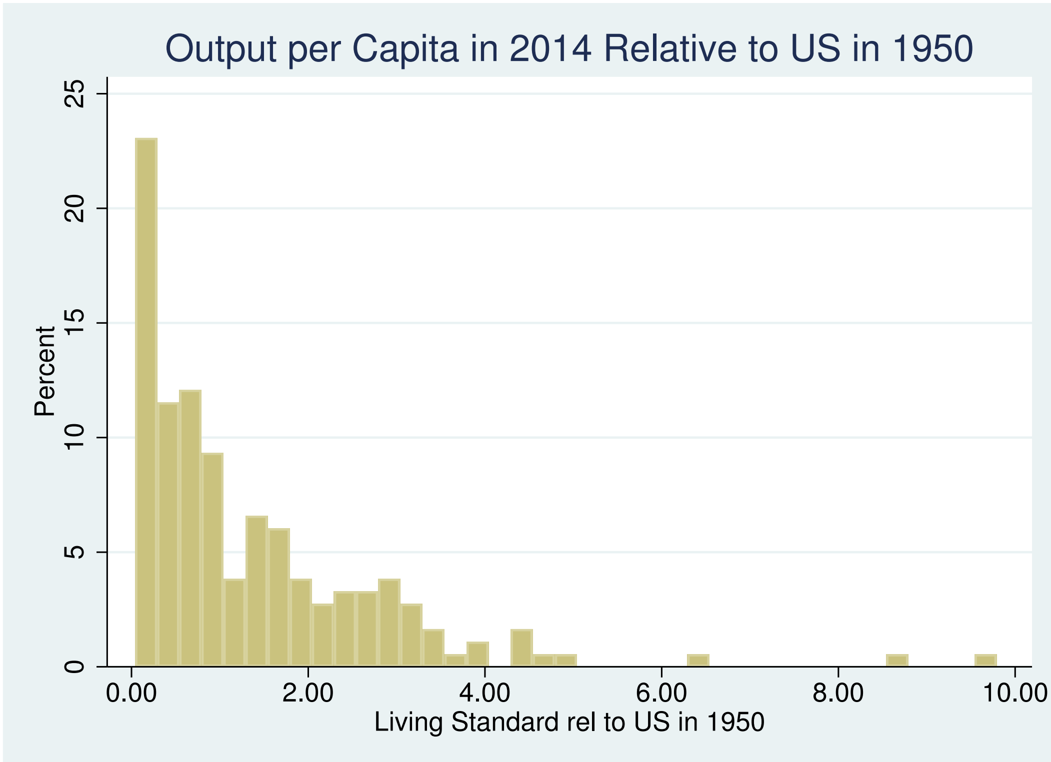

Figure 2.2: Per capita Output in 2014 Relative to US in 1950

is f_{2011}. Ireland's per capita GDP in 1990 was only 36 percent of that of the US in 2011. By 2011, Ireland's GDP per capita was 90 percent of that of the US. The catch-up was remarkable.

Consider this ratio:

$$Z_{j2014} = \frac{RGDP^e_{j2014}}{RGDP^e_{US1950}} \tag{2.14}$$

This is measuring the same thing as Q_{j2014}, the standard of living of country j relative to the US in 1950. The paper by Parente and Prescott (and the update I have prepared) use Q and not Z to measure progress across countries. The reason may be that the method used to construct $RGDP^e$ is very complex and not easy to understand. However, since the US is the numeraire, it is considered more reliable. In any case, the two are very close in most cases. In Table 2.2, the column labeled "Z" shows numbers quite close to those in the column labeled "Q". See Appendix 2.9 for more on this issue.

2.6 Other Data: World Bank and Maddison

The World Bank also has constructed a series for GDP that is comparable across countries and years. This data is freely obtainable at the "World Development Indicators" (WDI) website of the World Bank. The method is quite similar to that described here, but is not as complicated. Some find that a virtue, but others think it does not make some critical adjustments necessary to fully reflect purchasing power parity.

For a small set of countries there are data sets that go back to the nineteenth century. The most popular of these was constructed by Angus Maddison and periodically updated. This data has very few variables and mainly pertains to countries that have since become the richest of the world.[4]

2.7 Conclusion

We have just scratched the surface of the problem of international welfare comparison. The actual calculations are very involved, and no problem-free measures exist at present. Yet this should not stop us from using this data in empirical investigations.

[4]The data can be found at: MaddisonProject

2.8 Appendix A: The Dollar-Based PPP-ER

To construct F_j, the dollar-based PPP-ER for country j that we noted in (2.8), the PWT first operates at the category level of C, I, and G. I illustrate with C, the consumption category in which there are about $m = 120$ items. The ICP (International Comparison Program of the UN) reports an *average relative price* for each item in a country. Call these a^k_{ijt} where k stands for the category ($k = C, I, G$), i stands for the item (of which there are m within categories), j for the country (of which there are $n = 182$) and t for the year. We drop the "k" and "t" subscripts for the time being, to reduce clutter. The prices are relative to the United States' price and are defined as follows:

$$a_{ij} \equiv \frac{p_{ij}}{p_{iUS}} \tag{2.15}$$

For example, it i is the item "grain" and j is India, then a_{ij} would be an "exchange rate" – that is, a Rupee/$ ratio – between the US and India based on grain, not Big Macs.

The ICP also reports *relative quantities* q_{ijt} of each item in the C category. These are found from knowing the relative expenditure on item i in Country j and the US:

$$x_{ij} = \frac{X_{ij}}{X_{iUS}} \tag{2.16}$$

where X_{ijt} is the local-currency expenditure on item i in Country j in year t (again, we drop t for convenience). So the relative quantities are:

$$q_{ij} = \frac{x_{ij}}{a_{ij}} \tag{2.17}$$

These numbers will not have units. That is, in our example for India and grain, q_{ij} might be .23, meaning India consumes only 23 percent as much grain as the US.

Next, they use the data to construct a Laspeyres and a Paasche "price index" for every pair of countries j and h. These are calculated as follows:

$$fl_{jh} = \frac{\sum_{i=1}^{m} a_{ij} * q_{ih}}{\sum_{i=1}^{m} a_{ih} * q_{ih}} \tag{2.18}$$

$$fp_{jh} = \frac{\sum_{i=1}^{m} a_{ij} * q_{ij}}{\sum_{i=1}^{m} a_{ih} * q_{ij}} \tag{2.19}$$

Notice that the units of the two indices are the same: Curr$j$/Curr$h$, the price of currency h in terms of currency j, or how many units of currency j must be given up to get 1 unit of currency h. The numerators in (2.18) and (2.19) have units Curr$j$/\$ – like a_{ij} – and the denominator has units Curr$h$/\$. So the ratio is Curr$j$/Curr$h$. We are summing over i, the different items in the C category, so we are getting a *weighted average* of the a_{ij} "exchange rates" based the importance of grain, fruits, automobiles, medicine, beverages, etc. The only way that (2.18) and (2.19) differ is in the weights attached.

The geometric average of the two indices is called the Fisher Ideal Index:

$$f_{jh} = \sqrt{fl_{jh} * fp_{jh}} \tag{2.20}$$

This is the PPP exchange rate for the category C (Consumption) between country j and country h.[5] The PWT forms such PPP-ER's for all three categories, C, I, and G for all the countries and all seven benchmark years.[6] How many bilateral PPP exchange rates are there when there 182 countries? If there are n countries, then the formula is:

$$N = \frac{1}{2} n \left(n - 1\right) \tag{2.21}$$

For example, if there were 2 countries, there is 1 exchange rate. With 4 countries, there are 6. With $n = 182$, $N = 16,471$! Then, we have to multiply this by 3 (for the categories) and again by 7 (for the seven benchmark years). That makes $345,891$ different f_{jh}'s that are calculated with the basic data!

We now focus on the f_{jUS} rates (that is, $h = US$). Concentrate on the benchmark year of 2005. And let j be India again. Now, we have *three* $f_{IndiaUS}$ PPP rates, one for C, one for I, and one for G in 2005. Each was constructed using the method of (2.20). For any single category, say C,

[5] It is actually a bit more involved, and uses something called the GEKS index, but it is quite similar to the Fisher Index.

[6] In addition, they do it for a few export and import categories, but I ignore those here.

Equation (2.18) shows that the Laspeyres index of price relatives for India is just the simple average of those price relatives, since $a_{iUS} = q_{iUS} = 1$:

$$fl_{IndiaUS} = \sum_{i=1}^{m} \left(\frac{1}{m}\right) a_{iIndia} \tag{2.22}$$

The Paasche index, however, is a *weighted* average of the price relatives:

$$fp_{IndiaUS} = \sum_{i=1}^{m} \theta_i a_{iIndia} \tag{2.23}$$

where the weights are the shares of the relative expenditure on item i in India:

$$\theta_i = \frac{q_{iIndia}}{\sum q_{iIndia}} \tag{2.24}$$

which have to sum to 1. Plug (2.22) and (2.23) into (2.20) to get the PPP-ER-C rate for India and the US. To find PPP-ER-I and PPP-ER-G, similar procedures are followed. That gives the three PPP-ER's for 2005 for one country, India. We find all *sets of three* (j/US) PPP-ER's this way for all 182 countries in 2005. Then, repeat for the other six benchmark years. It sounds like a lot of work, but once programmed, computers can do this work in seconds.

Each benchmark year is self-contained in that the PPP-ER grids are valid only for that year. That is, $f_{USUS} = 1$ in *all* of the benchmark years. Assume that in 1990 in the C category $f_{NewZealandUS} = 1.10$ and $f_{NorwayUS} = 7.14$. That means that Purchasing Power Parity would require that it take 1.10 New Zealand dollars to buy one US dollar (it actually took 1.68); and 7.14 Norwegian krone to buy a dollar in 1990 (it actually took 6.26). Those rates would equalize the value of \$100 in the purchase of consumption goods in the three countries in 1990. In 2005, $f_{USUS} = 1$ again, but the other rates might well be different. That is, within benchmark years, they are relative to the US.

To link the years, the f_{jUS} rates are multiplied by the US price index for that year Π_t. We defined Π_t in Section 2.3 as the relative price of Big Macs over time in the US. Now we use the general price index called the "GDP deflator".

In 2005 $\Pi = 1$. That is, the base year for inflation calculations in the US is 2005 so $\Pi_{2005} = 1$. That means for the benchmark year of 2005, multiplying all of the f_{jUS} rates by Π_t has no effect. For every other benchmark year, however, it does matter. For example, if $\Pi_{1990} = .812$, given the numbers above, then $f_{NewZealandUS} * \Pi_{1990} = .893 NZ/\$$.

Next, the PWT interpolates and extrapolates all the PPP category rates for every year from 1950 to 2014, using data on the price levels for each of the two countries. In this way, they have a complete set of PPP exchange rates for every year for each of the three categories, normalized on the US price level.

The last step is to combine or aggregate the three category PPP-ER's into one PPP-ER for each country in each year.

Above, we saw that the ICP collects data on *domestic-currency expenditure* X_{ij} on each item in each category in each country. If we add the expenditure on all the items in category C we can call the result $X_j^C = \sum_{i=1}^m X_{ij}^C$. Conceptually, note that $X_j^C = \mathbf{p'}_{ij}\mathbf{q}_{ij}$, where $\mathbf{q}_{ij}$ is a vector of the quantities produced of each item in category C in country j. Therefore, X_j^C, X_j^I, and X_j^G are the *total money amounts* that people spend in their own currencies on, respectively, the C category, the I category, and the G category. Divide each of these by f_{jUS} to get:

$$Q_j^k = X_j^k / f_{jUS}^k = \mathbf{p'}_{iUS}\mathbf{q}_{ij} \quad (k = C, I, G) \tag{2.25}$$

Each Q_j^k is the expenditure for category k in Country j *expressed in US dollars* of the current year and f_{jUS}^k is the PPP-ER for category k in country j. The vector notation suggests that it is *as if* we knew the prices of the goods in *dollars* not local currency.

Now add up all the Q_j^k in one country to get GDP of Country j measured in US dollars. That is:

$$Y_j^{\$} = \sum_k Q_j^k \tag{2.26}$$

Note that the k drops out from the left-hand side because we are adding up over the three categories for country j.

Finally, we find the overall PPP-ER in dollars, which we call F_j reported in (2.8) for Country j as:

$$F_j = \frac{Y_j}{Y_j^{\$}} \tag{2.27}$$

where, as defined in (2.1), Y_j is GDP in local currency prices.[7]

2.9 Appendix B: The Method of Parente and Prescott

Parente and Prescott – and my update that we go over in class – use the PWT data for 102 countries to construct a consistent matrix to compare real GDP per capita. The first step, for any year t, is to form the ratio of $CGDP^e$ for Country j and the US:

$$L_{jt} = \frac{CGDP^e_{jt}}{CGDP^e_{USt}} \tag{2.28}$$

Although they refer to this ratio as a country's *year-t relative wealth*, it is better to call it a nation's *relative income* or *relative production* (remember that the value of income and production are equal). In Column 5 of Table 2.2 we show the values for Ireland of L_{Iret} for t going from 1990 to 2014. To take an example, $L_{Ire,1997} = 0.64$: this means that Ireland in 1997 produced only 64% of the "average good" compared to the US in 1997.

To construct a measure that is comparable across time as well as space, each L_{jt} is multiplied by the following factor:

$$f_{t2011} = \frac{RGDP^e_{USt}}{RGDP^e_{US2011}} \tag{2.29}$$

This is the adjustment factor for each year. This ratio is the value of real GDP produced per person in the US in year t, relative to the amount produced per person in the US in 2011 (the base year). For example, $f_{2004} = .75$

[7]In reality, the construction is a lot more complicated than outlined here. In particular, Equation (2.26) is too simple. Instead, the PWT uses something called "reference prices" to add up the domestic outputs, q_{ij}. Here, I use dollar prices in the US in year t. Reference prices are weighted averages of dollar category prices in all the countries.

means that 25% less of the average good was produced per person in the US in 2004 compared to 2011.

The multiplication of L_{jt} by f_t yields a measure of real per capita GDP in year t for Country j that is *relative to the US in 2011 (the base year)*. In other words, the product $L_{jt} \times f_t$ is measuring exactly the same thing that is measured by Z in (2.14). We can show this by thinking of the summations as the multiplication of two vectors. Thus:

$$Q = L \times f = \frac{\mathbf{p}_t^{'} \bullet \mathbf{q}_{jt}}{\mathbf{p}_t^{'} \bullet \mathbf{q}_{USt}} \times \frac{\mathbf{p}_{2011}^{'} \bullet \mathbf{q}_{USt}}{\mathbf{p}_{2011}^{'} \bullet \mathbf{q}_{US2011}} = \frac{\mathbf{q}_{j,t}}{\mathbf{q}_{US,2011}} \tag{2.30}$$

That is, the product measures the quantity of the typical good in Country j in year t relative to the quantity of the typical good produced in the US in the base year, 2011.

The measures Z and Q are, however, slightly different. Columns (6) – (8) of Table 2.2 present the data for the construction of the Parente and Prescott measure for Ireland from 1990 to 2014 (again, the reference year is the base year 2011). The constructs for the US and Ireland for L_{jt}, f_t, and $Q = L_{jt} \times f_t$ are shown. The last column shows Z_t for Ireland. The last two columns are very close in magnitude.

Chapter 3

Principal Eras of Economic History

3.1 Introduction

Human societies have been hit by both slow and precipitous changes to population, technology, and institutions since the dawn of time. Modern economies are vastly more complex than early societies, yet we can learn valuable lessons from the study of transitions from one economic era to the next.

This chapter simply sets out, in extremely condensed form, the basic progression of the economic eras that have characterized the world (mainly, the western, industrialized world). This will help place in context the work of Douglass North (*Structure and Change in Economic History*, Chapter 7, 1981) that explains the transition to agriculture from hunting and gathering, the first economic revolution.

3.2 Eras

3.2.1 Hunting and Gathering

1 Million BCE to 8,000 BCE

Small bands of people spread throughout the world. Production from hunting wild animals and harvesting natural crops. Extensive use of large amounts of land. Population grows very slowly. Slow, but perceptible, development of new technologies. Government is tribal. Property rights are Common Property, but territorial.

3.2.2 Neolithic Agriculture

8,000 BCE to 4,000 BCE

Bands become sedentary and employ agriculture primarily. Groups become larger. Faster population growth. Trade and division of labor increase. Technology progresses faster. Government remains tribal; property rights are Communal, but exclude outsiders.

3.2.3 River Empires

4,000 BCE to 800 BCE

Rise and Fall of the great riverine Empires: Sumeria, Egypt, Babylonia. Great expansion of trade in the Mediterranean. Population may have grown at very fast rates. Technological advances were far greater than before. Written language appears. Government was autocratic, relying on religion; property rights associated with the deity and ruler.

3.2.4 Classical

800 BCE to 400 CE

Greece and Rome. Coinage appears. Advances in legal systems and the arts. Commerce expands, as do living standards. Population may have been very large. Technology stifled for commercial purposes. Government autocratic; property rights well-defined for certain groups.

3.2.5 Dark Ages

400 CE to 1,000 CE

Fall of Rome and retreat to Feudalistic self-sufficiency in manors. Towns disappear and trade declines greatly. Population and living standards fall considerably. Monastic learning; few advances in technology. China advances beyond Europe. Government chaotic, based on force; property rights very insecure.

3.2.6 Revival

1,000 CE to 1350 CE

A revival centered on Northern Italy. Increasingly fast population growth leads to dense population and increasing specialization and commercial revival. Migrations into the Baltics and the East. Technology begins to advance more rapidly. Mongol invasions halt Chinese development. Governments diverse around the world; property rights well defined in certain parts of Europe. Bourgeoisie arises.

3.2.7 Black Death

1350 to 1500

War, famine, Black Death lead to a general retrogression in population and real income. Population declines by 30% to 50% in Europe. Commerce contracts, self-sufficiency returns, as production retreats to the country. Technology stagnant, except for war technology. Governments defensive; property rights show no progress.

3.2.8 Renaissance and Enlightenment

1500 to 1780

Renaissance to Enlightenment and Age of Discovery. Expansion of trade, population, and knowledge — again. Economic center moves to North Atlantic (Holland, England, Belgium) and away from Northern Italy. Emergence of nation-states and political integration. Technology begins to make important strides. Governments become more central and larger; property rights differ greatly in Europe and the world.

3.2.9 Industrial Revolution and Expansion

1780 to 1900

Industrial Revolution and expansion. Spread of industrial technology around the world. Colonialism. Migrations and economic integration. Growth becomes fast and routine in some nations. Population growth in Europe is astounding. Technology drives unprecedented growth from Europe to North America. Governments become increasingly democratic; property rights in some places become secure for the first time.

3.2.10 Scientific Revolution and Modern Growth

1900 to Present

Scientific Revolution. Increase in living standards in some nations based on application of science to production. Appearance of huge disparities in income across nations. Population growth falls dramatically in rich countries. Technology proceeds extremely rapidly. In developed nations, governments increasingly democratic; property rights increasingly secure.

3.3 Conclusion

History can provide important lessons for economic development, and it is useful to be aware of the kinds of transitions that have already taken place. The current situation is novel in the sense that some areas have gotten far ahead of others in terms of technology and living standards. The challenges of dealing with this state of affairs are clear, most especially politically.

Advantages also exist, however, since there is hope that developing nations can find ways to take advantage of the new techniques of both production and political organization.

Chapter 4

Rates of Growth

By: John McDermott (Economics 705)

4.1 Introduction

This chapter sets out some important technical results on the meaning and manipulation of growth rates and interest rates in both discrete time and continuous time. Without a basic understanding of the movement of variables over time, and their changes over very short intervals, it would be impossible to think clearly about economic development.

4.2 Growth Rates: Discrete and Continuous

In economic dynamics, we may use one of two types of analysis: *discrete* or *continuous*.

In the discrete framework, the variables of interest take on a single value per time period (usually, a year). As time advances from Year t to Year $t+1$ to Year $t+2$, etc., the variable's value changes: it jumps discretely at the end of each period to its new value to begin the next period. *Within* periods, the variable does not change at all.

In the continuous framework, the variables are constantly changing at a steady rate through time t.

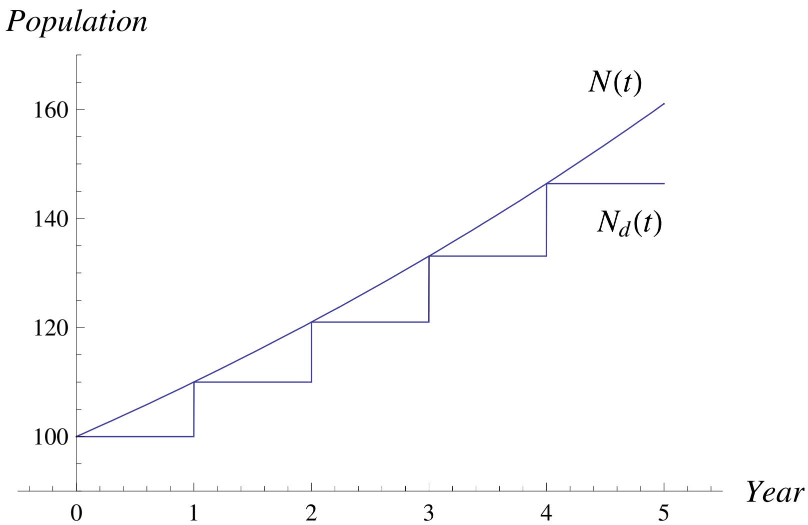

Figure 4.1: Population Over Time: Discrete and Continuous

We illustrate both types of change or growth with the *population of Europe*, which we shall refer to as N. The centralizing concept of growth is very simple: the *annual percentage change* in the variable in question. This is also the principal concept for interest and capital appreciation and we shall discuss these concepts as well. A key word here is "annual": all rates of interest, growth, or change must be defined fundamentally in terms of a precise time dimension. It is almost always a year.

Although we will use real data shortly, assume at first that population is growing at about 10% per year. **Figure** 4.1 shows the paths of N under both discrete and continuous growth for the first five years, assuming it began at 100 (that is, 100 million people). The actual growth of population in Europe was far smaller.

The smooth curve labeled $N(t)$ is the continuous case. The choppy curve labeled $N_d(t)$ reflects the discrete case. Notice that the two paths both begin at $N(0) = N_0 = 100\,\text{million}$.

The choppy path is the easiest to explain, so we begin there. The formula for N after one year is given by the basic application of percentage change:

$$N_1 = N_0\,(1 + n_d) = 100(1.1) = 110. \tag{4.1}$$

In the above expression, n_d is the *geometric growth rate*, which here is .10 or 10%. To find the value for subsequent years, we may apply the above formula repeatedly. For example, for 2 years it is:

$$N_2 = N_1\,(1 + n_d) = N_0\,(1 + n_d)^2 = 100(1.1)^2 = 121. \tag{4.2}$$

For t years, where t is any *integer*, we find that population is given by:

$$N_t = N_0\,(1 + n_d)^{\,t}. \tag{4.3}$$

We can always calculate the geometric growth rate if we have data for any two adjacent years. This formula is well-known and follows from (4.3):

$$n_d = \frac{N_t - N_{t-1}}{N_{t-1}}\ . \tag{4.4}$$

Now consider the smooth growth path also shown in Figure 4.1. This corresponds to *exponential* or *continuous growth*. In terms of interest, it corresponds to *continuously compounded interest*. The formula for the smooth line is given by:

$$N(t) = N(0)e^{nt} = N_0\mathrm{Exp}(nt)\ , \tag{4.5}$$

where n is the *exponential growth rate*. The rates n and n_d are not the same, although they are closely related and one can be found from the other. The variable e is simply a number. It is a natural constant, like π, and it is given by $e = 2.71828$....... This number never repeats and is not completely known. It is sometimes clearer to write the exponential function as $\mathrm{Exp}(nt)$, but the meaning is exactly the same as e^{nt}.

The number e is rather mysterious. It can be derived in several different ways and arises in different contexts in nature. To illustrate one way of finding it, we consider the case of compound interest, which is just another growth process. Let us assume that the interest rate r is 100 percent; that is, that $r = 1.0$. So, if you had \$$A$ on January 1, 2018 and put it in the bank at simple annual interest, you would have \$$2A$ on January 1, 2019. Using the formula (4.1), we could write $A_{t+1} = A_t\,(1+r) = A_t\,(1+1) = 2A_t$, where A_t is the amount you put in at the beginning of year t and A_{t+1} is the amount you have at the beginning of year $t+1$.

Now, instead of letting this process go forward over several years, let us do something else. Let's *compound* the interest *within one year*. First define the sub-period: a month, a week, a day, etc. Let's take a month. That means that each month the bank pays you $\frac{r}{12} = \frac{1}{12}$ percent interest, so if you began with $A_t = \$1.00$, after one *month* you have $\$\left(1 + \frac{1}{12}\right)$. That is now the principal, so that after two months you have:

$$\$\left(1 + \frac{1}{12}\right) * \left(1 + \frac{1}{12}\right) \tag{4.6}$$

and after a year you have:

$$A_{t+1} = \$\left(1 + \frac{1}{12}\right)^{12} = \$2.61304 \tag{4.7}$$

Notice that this is a lot better than the \$2.00 you'd get from a simple annual interest rate of 100 percent. If the interest were compounded *daily*, we would have $\$\left(1 + \frac{1}{365}\right)^{365} = \$2.71457$ after one year. It's bigger, but not much bigger. What happens if we let the sub-unit get very, very small, so the number of units in a year – call it v – gets very, very big? This is what happens:

$$\lim_{v\to\infty}\left(1 + \frac{1}{v}\right)^{v} = e = 2.718281828.... \tag{4.8}$$

Remarkable! It converges to the natural constant e. You can easily test this with a calculator by plugging in a number like $v = 10{,}000$. It will be very close to the e on your calculator.

So, if you had \$$A_t$ to start with, if interest were *continuously compounded* at 100 percent per year (the "per year" is very important) you would have $\$A_{t+1} = \$A_t e$ at the end of the year. And, if you kept it in the bank for 2 years, you would have $\$A_{t+1} = \$A_t e^2$ at the end. Why? Because you end the first year with Ae, which is then the beginning amount for the second year, so you have $(Ae)\,e = Ae^2$ after two years.

In fact, for any length of time t, whether or not it is an integer, if the interest rate is 100 percent, you end up with $\$Ae^t$ after t years.

Finally, what if the interest rate is a more reasonable number like $r = .05$? Then we get:[1]

$$A\,(t) = A\,(0)\,e^{rt} \tag{4.9}$$

This looks a lot like (4.5). That is because we can think of population growing in the same way that money in the bank grows. Instead of an interest rate r, though, we have the rate of population growth n.

The exponential growth rate n that corresponds to the geometric yearly rate n_d is always smaller than n_d. How are n_d and $n$related? Using (4.3) and (4.5) and letting $t = 0$ we note that the following must be satisfied:

$$\frac{N\,(1)}{N\,(0)} = (1 + n_d) = e^n \tag{4.10}$$

If we knew $n$we could find n_d easily: $n_d = e^n - 1$. But what if we know n_d? How do we find n? For that we need the natural log function, discuss below.

4.3 Exponential Growth Rates in Practice

We go through the bother of discussing exponential rates because they are much easier to work with than geometric rates. For this section, recall from high-school math that $x^2 x^3 = x^5$ and that $x^3/x^2 = x$.

Consider, for example, the progress of per capita output $y = \frac{Y}{N}$ over time (here Y is GDP), something that we are very concerned about. Assume that Y is growing exponentially at the rate g while N, as before, is growing at the exponential rate n. Then it is straightforward to show that y is growing at the rate $g - n$. Here is how to show it:

$$y(t) = \frac{Y(t)}{N(t)} = \frac{Y_0 e^{gt}}{N_0 e^{nt}} = y_0 e^{(g-n)t} = y_0 \mathrm{Exp}[(g-n)t]\ . \tag{4.11}$$

[1] See Appendix A to this chapter, to see how to derive these two results.

We infer by inspection that the rate of change of y is $(g - n)$.

Consider now revenue $R = PQ$: revenue is price times sales. You should be able to work out that the growth rate of revenue is equal to the growth rate of the price *plus* (not times) the growth rate of sales.

Another example concerns a popular production function. Let us say that output depends only on capital and that there are diminishing returns: $Y = K^\alpha$, where $\alpha < 1$. If capital is growing at the rate g_K, how fast is output Y growing? We derive it as follows.

$$Y(t) = K(t)^\alpha = \left(K_0 e^{g_K t}\right)^\alpha = K_0{}^\alpha \left(e^{g_K t}\right)^\alpha = Y_0 e^{\alpha g_K t}. \qquad (4.12)$$

From this we see that the growth rate of Y is given by $g = \alpha g_K$. This derivation relies on the result that $\left(x^2\right)^3 = x^6$.

You should be able to show that the growth rate of $Z = \left(\frac{K}{N}\right)^\beta$ is given by: $g_Z = \beta\left(g_K - n\right)$.

One final problem: find the growth rate of Y if the production function is $Y = K^\alpha L^\gamma$ where L is workers, which differs from population. That is, find g in terms of the growth rates of capital g_K and workers g_L (which may equal n even though $N < L$).

These results are even more important than they might seem because they generalize to *instantaneous growth rates*, no matter what the underlying process that is generating the change. We discuss this in more detail below. Next, however, we discuss the natural log function.

4.4 Natural Logs

Everyone knows what a *square root* is. But defining it out loud can be slightly difficult. The reason for that is, I think, that all a square root does is *undo* a square! Without the square of a number, the square root is meaningless. To see this more formally, define the following function that squares a number:

$$S(x) = x * x = x^2 \ , \tag{4.13}$$

where we may call $S(x)$ the "square function". That is, $S(5) = 25$. Simple enough.

Now define the "square root function" $R(x)$. This function just *reverses* or *undoes* the square function. That is, $R(25) = 5$. It is hard to write down a general function the way we wrote (4.13) above. Perhaps the most informative way to write it is as follows:

$$S(R(x)) = x \quad and \quad R(S(x)) = x. \tag{4.14}$$

That may look odd but it just says what we all know: $\left(\sqrt{x}\right)^2 = x$ and $\sqrt{x^2} = x$.

The reason to bring this up is that the *natural log function* $\ln(x)$ is much the same as the square root. It simply *undoes* the exponential function. That is:

$$\ln\left(e^x\right) = x \text{ and } e^{\ln(x)} = x \ . \tag{4.15}$$

One very important property of the log function is that:

$$\ln(AB) = \ln A + \ln B. \tag{4.16}$$

Here is how to show (4.16). Use (4.15) to express $AB = e^{\ln A} e^{\ln B}$. But the latter can be written as $e^{\ln A + \ln B}$. Now take the log of the first and last expressions in that sequence. They must be equal, which proves (4.16). It also follows that $\ln(A/B) = \ln A - \ln B$, another very useful result.

Now we can show how to find n_d if we know n. From (4.10), take logs of both sides to get:

$$\ln\left(e^n\right) = n = \ln\left(1 + n_d\right) \tag{4.17}$$

So if $n_d = .10$, then $n = .09531$. An exponential growth rate of .09531 is sufficient to keep up with a simple annual growth rate of .10, which is what is shown in Figure 4.1.

We illustrate the use of the natural log in visualizing the population of Europe in the next section.

4.5 The Population of Europe

Our best estimates show that Europe's population was 81 million in 1500 AD and rose to 728 million in 2000. We shall simply *assume* or *impose* exponential growth with a constant growth rate from the beginning to the end of the 500 year path. This means that we assume the following is satisfied:

$$N_{2000} = N_{1500}e^{nt}. \tag{4.18}$$

Notice that we know everything about (4.18) except the value of n. Thus, $t = 500$, $N_{1500} = 81$, and $N_{2000} = 728$. To find n, we substitute in the values, then take the natural log of both sides of (4.18) to get:

$$\begin{aligned} \ln 728 &= \ln 81 + n500 \implies \\ & \quad n = \frac{1}{500}(\ln 728 - \ln 81) = .0043917 \text{ or } .439\% \,. \end{aligned} \tag{4.19}$$

In other words, the population of Europe grew, *on average*, at less than $\frac{1}{2}\%$
.

Figure 4.2 shows our hypothesized, exponential path for population. Of course it was not that smooth: war, famine, and disease still wreaked havoc on the European population even after 1500. Only the first and last points are "known": the others are generated to make the path smooth.

Figure 4.3 shows the path of the *natural log of the population* over this time period (again, our hypothesized path). It is given by

$$\ln N = \ln 81 + .00439t$$

The interesting thing here is that the *natural log* of a variable that is growing exponentially in nature is a *straight line*. It is amazing how many economic

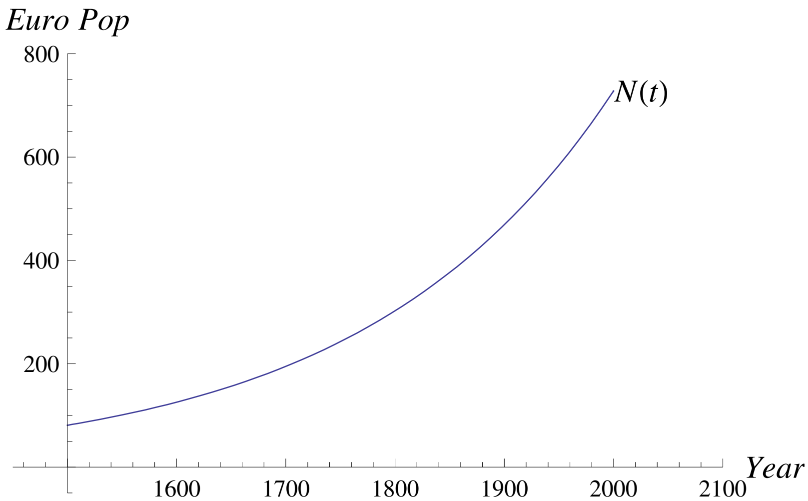

Figure 4.2: Population of Europe

time series when transformed into the natural logarithm (like the natural log of US GDP per capita) are straight lines over many years.

4.6 Negative Exponential Growth

Nothing prevents the growth rate from having a negative value. For example, if population were growing faster than GDP, the growth rate of y would be negative. You can see that this is a possibility from Equation (4.11). Assume that $g = .02$ and $n = .04$, which is large, but not out of the question. Figure 4.4 shows what the path of y through time would look like. It is amazing how quickly $y(t)$ falls to near-zero at only a 2% annual decline. Luckily, such long-run declines are rare, even with war and very unstable government, although they are more common than you probably think.

Another case is depreciation of capital, like a house or car or factory. That is, wear and tear on machines makes it gradually decline in ability to produce. Let δ be the rate of depreciation; say, $\delta = .06$, so that machines wear out at the rate of 6 percent a year. If this is a continuous (exponential) rate, we express this as:

$$K\left(t\right) = K\left(0\right) e^{-\delta t} \tag{4.20}$$

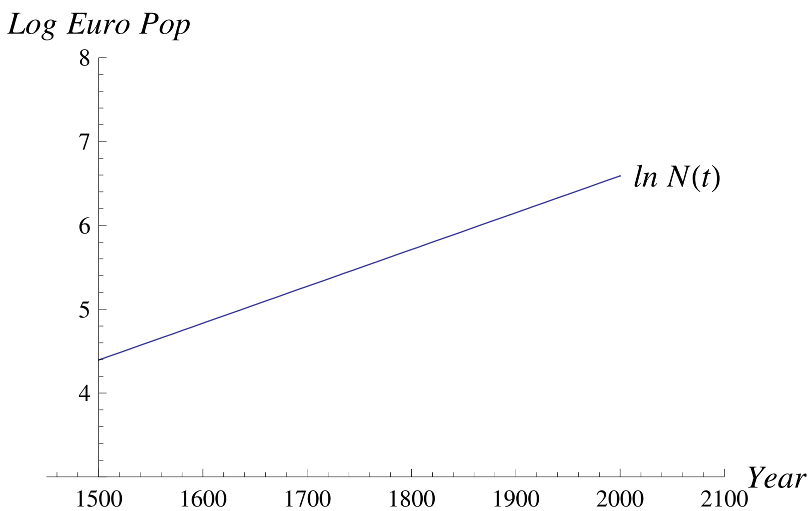

Figure 4.3: Log of European Population

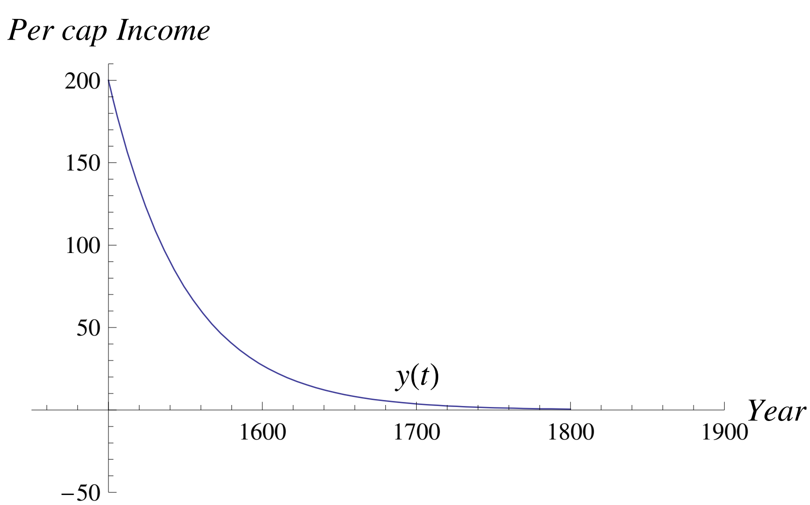

Figure 4.4: Negative Exponential Growth

4.7 Instantaneous Growth Rates

So far, we have treated rates of change over very long intervals, seeking to find the average annual growth rate – either discrete (also called geometric) or exponential (also called continuous). Now we focus on a smaller time period so that the results pertain to *any* variable moving through time.

First, let's consider a *yearly percentage change* in the population. From (4.4) and (4.19) – with $t = 1$ – we may say that the two rates n_d and n are *approximately* equal. Let us write them as follows:

$$n_d = \frac{N_{1996} - N_{1995}}{N_{1995}} = \frac{\Delta N}{N} \approx n = \ln N_{1996} - \ln N_{1995} = \Delta \ln N \ . \quad (4.21)$$

The symbol "Δ" means "change in" over a specific time period. The squiggly equals sign " ≈ " means "approximately equal to".

What if we were interested in the *yearly* percentage change over a smaller interval, like 6 months? Again, everything is based on a yearly rate. This is very important. To find the answer, we would "pro-rate" the percentage change by dividing through by the *fraction of the year* over which the change takes place. We call this *fraction* "Δt". Thus, we re-write (4.21) as:

$$n_d = \frac{\Delta N}{\Delta t}\frac{1}{N} \approx n = \frac{\Delta \ln N}{\Delta t} \ . \quad (4.22)$$

The above is very general, since for a year $\Delta t = 1$. For 1 month, $\Delta t = \frac{1}{12}$; for a day, $\Delta t = \frac{1}{365}$. The change in population is accordingly measured only over the month or day as the case may be. By dividing this way, we keep expressing the change on an annual basis.

Although we cannot show it here, something very interesting happens as the time interval gets short: the approximation gets better. For a very short – infinitesimally short – interval, the two are exactly the same! We use the symbol "d" in place of "Δ" to refer to this extremely short time interval. Thus, we may now write:

$$n = \frac{dN}{dt}\frac{1}{N} = \frac{d \ln N}{dt} \ . \quad (4.23)$$

Perhaps an example will help clarify the basic idea. Let's say that at the very beginning of the day of July 16, 2005 the population of the Czech Republic was $N = 10.24$ million. On that day, the net increase in the population was 420 people (not millions!). First, note that the first term in (4.23) can be written as:

$$n = \left(\frac{420}{\frac{1}{365}}\right)\left(\frac{1}{10.24 * 10^6}\right) = 0.0149707$$

How do we interpret this? It is the percentage by which the Czech population *would have grown over the year*, if the rate of 420/day had continued. Again, note that the *year* is the key. This is about 1.50% growth rate for the Czech population.

The first ratio of (4.23), $\frac{dN}{dt}$, is called the *time derivative* of the path of population and it is very important in growth theory. Notice that it is closely related to the *growth rate* or *percentage rate of change*, n. They differ only by the factor N. Often, we use the dot notation for the time derivative:

$$\dot{N} = \frac{dN}{dt}$$

We can then write (4.23) as:

$$n = \frac{\dot{N}}{N} \implies \dot{N} = nN \; . \tag{4.24}$$

Again, the meaning of $\dot{N}$ is the absolute change in N over a small interval, *expressed on a yearly basis.*

What about the second expression in (4.23), the log form? The change in the log of N can be written as $\ln[(N + \Delta N)/N]$. This uses (4.15) above. Here is the math for calculating the growth rate using logs:

$$n_2 = \ln\left(\frac{10.24 * 10^6 + 420}{10.24 * 10^6}\right) * 365 = 0.0149704$$

Notice that the two rates are extremely close. This is because our time interval – a day – is very short. If we did a calculation for a *minute*, the two numbers would be even closer.

Although I have used population here, the ideas are applicable to any economic time series, such as GDP, capital, or the price level. In the case of capital K for example, we can use the example of depreciation from above to write:

$$K(t) = K(0)\, e^{-\delta t} \Longleftrightarrow \dot{K} = -\delta K \Longleftrightarrow \frac{\dot{K}}{K} = -\delta \Longleftrightarrow g_K = -\delta \tag{4.25}$$

It is easy to go back and forth between growth rates and time derivatives. In the next chapter we will employ these techniques to discuss the fundamental model of economic growth.

4.8 Related Concepts

4.8.1 Present Value

Suppose you will receive \$$B$ in year T. What is that worth *today?* You could say it is worth the amount \$$A$, such that, with interest, \$$A$ will grow to \$$B$ in T years. As before, we may analyze the question in either discrete or continuous time.

In discrete time, A must satisfy:

$$A(1+R)^T = B \implies A = \frac{B}{(1+R)^T} \;. \tag{4.26}$$

In continuous time, A must satisfy:

$$Ae^{RT} = B \implies A = Be^{-RT} \;. \tag{4.27}$$

Here is a simple problem. Find the present value of \$100 to be received in 10 years, if $R = .03$. Find both the continuous-time value and the discrete-time value.

According to (4.26): $A = \frac{100}{(1.03)^{10}} = \$74.41$.

According to (4.27), we find: $A = 100e^{-.03*10} = \$74.08$

At very low interest rates, the two are very similar.

4.8.2 Annuity

Let's say you had $\$A$ today. What *constant amount* could you consume *each year forever* with that amount? The answer is what we call the "annuity value" of the principal.

To find the answer, we note that in continuous time, the initial amount must equal the integral of the present value of each year's constant consumption, C. That is:

$$A = \int_0^\infty Ce^{-Rt}dt \implies C = RA \tag{4.28}$$

The formula is very simple and intuitive: you can consume the interest on the asset forever.

In discrete time, the condition is still the sum of present values, which leads to:

$$A = \sum_{t=0}^{\infty} C\left(\frac{1}{1+R}\right)^t \implies C = \left(\frac{R}{1+R}\right)A \tag{4.29}$$

If Jane has an inheritance \$250,000, how much could she consume forever given that the interest rate is 10%? Find the answer in both continuous and discrete terms.

In continuous terms:

$$C = .10 * 250,000 = 25,000 \tag{4.30}$$

In discrete terms:

$$C = \left(\frac{.10}{1.10}\right)250,000 = 22,727.30\ . \tag{4.31}$$

4.9 Conclusion

These are important techniques for use in all advanced economics and finance courses. After working with them for a while, it becomes much easier to use them and understand why they are so valuable.

Appendix A: Exponential function with $r \neq 1$ and $t \neq 1$

We first address the issue that t may not be one year. The original monthly compounding formula for one year is given by (4.7). But we could easily write this as:

$$W = \left(1 + \frac{1}{12}\right)^{12*t} \tag{4.32}$$

where t is measured as a multiple or fraction of a year. So if $t = 1$, the money is left in the bank for one year. If $t = 2$ it is left for 2 years. If $t = 1/2$, it is left for 6 months. It can be bigger than 1 or smaller than 1 but it must be in multiples of 1/12. Or, in multiples of $\frac{1}{v}$ where v is the number of times the compounding takes place within a year. So, the formula (4.8) now becomes:

$$\lim_{v \to \infty} \left[\left(1 + \frac{1}{v}\right)^{v}\right]^{t} = e^{t} \tag{4.33}$$

As v gets very large, t becomes continuous since every number is a multiple of $\frac{1}{v}$.

To extend the formula to the case in which r is not 1.0, write (4.32) as:

$$Z = \left(1 + \frac{r}{12}\right)^{12*t} \tag{4.34}$$

That is, if $r = .10$ is the annual interest rate, then you receive $\frac{r}{12} = .00833$ each month. For the general case, it is:

$$Z = \left(1 + \frac{r}{v}\right)^{vt} = \left[\left(1 + \frac{r}{v}\right)^{v/r}\right]^{rt} \tag{4.35}$$

Now take the limit again, to get:

$$\lim_{v \to \infty} Z = e^{rt} \tag{4.36}$$

which is what we set out to show.

Chapter 5

The Neoclassical Growth Model

5.1 Introduction

This chapter sets out a basic version of Solow's (1956) classic paper that has influenced an entire generation of economists and policy makers. This model relies on the accumulation of physical capital to explain how living standards change over time. The fact that capital's marginal productivity declines as more is accumulated, however, means that an economy with a constant saving rate achieves a steady-state equilibrium with no growth. Constantly rising living standards are possible only with continuous technological change. Without such technological advance, growth is only temporary, or transitional.

5.2 Equations of Change

At the center of the neoclassical model is the *stock* of physical capital, K. The change in the capital stock per unit of time over a small time interval is given by the following equation:

$$\dot{K} = sY(t) - \delta K(t) \ . \tag{5.1}$$

As in the last chapter, we use the "dot" notation to refer to the change in the variable per unit of time:

$$\dot{K} = \frac{dK}{dt} \tag{5.2}$$

On the right side of (5.1), s is the *saving rate*, δ is the *depreciation rate*, and Y is GNP or national income. The way to interpret (5.1) is that change of the capital stock K per unit time is equal to the difference between the amount saved and the amount of wear and tear. Total national saving (all of which is assumed to go to new capital) is sY, the saving rate times total income. For example, assume that $\delta = 0$, the saving rate is $s = .20$, and $Y = \frac{400 goods}{Year}$. Then the capital stock increases by 80 units per year.

Physical capital *does* depreciate, however, so we need the second term in (5.1) take account of this process of wear and tear. If there were no saving ($s = 0$) capital would wear out at a rate proportional to its size. Dividing both sides of (5.1) by K and using (5.2) and (**??**) shows that, if $s = 0$, then K capital would follow a *negative exponential process*. The absolute decline in capital from depreciation is not constant, even though the rate of decline is constant. For example, if the capital stock were 100 units, and the depreciation rate were $\delta = .08$, then over one unit of time 8 units of capital simply disappears, totally worn out. In the following period, if there has been no saving and investment, then 7.36 units disappears ($92 * 0.08 = 7.36$).

Labor, L, is changing through time, too. We assume that it is a simple positive exponential process as we did in Chapter 4. Moreover, we assume that the labor force is growing at the same rate n as population:

$$L(t) = L_0 e^{n*t}. \tag{5.3}$$

We saw in Chapter 4 that we may express the growth rate:

$$n = \frac{\dot{L}}{L} \tag{5.4}$$

from which we can find the change per unit of time of L:

$$\dot{L} = nL. \tag{5.5}$$

Notice that the change in L $\left(\dot{L}\right)$ is actually rising over time, even though its *growth rate* is constant at the rate n.

5.3 Production

Why do people bother to accumulate capital? Because it is useful for producing goods that may be consumed. We use the following *production function:*

$$Y = K^{\alpha}(AhL)^{1-\alpha} \tag{5.6}$$

This is called the Cobb-Douglas production function and is very widely used in growth theory and other production theory. Here, A stands for "labor-improving technology", and h stands for human capital or education that improves each worker. This function shows constant returns to scale in K and AhL together, but diminishing marginal product to either input separately (the appendix defines and derives the marginal product of capital). For the time being, let us assume that $h = 1$.

Technology, A is growing at the rate g:

$$A\left(t\right) = A\left(0\right)e^{g*t} \tag{5.7}$$

Define B as follows:

$$B \equiv A^{1-\alpha} \tag{5.8}$$

We will refer to B as "technology", too. We do this because we can then write the production function as

$$Y = BK^{\alpha}L^{1-\alpha}$$

since we have assumed $h = 1$. This is the form in which it is often written, but I began with Eq. (5.6) to emphasize two features: (1) human capital improves labor; (2) technology must be labor-saving – as is A – to be consistent with balanced growth (as defined later).

We are most interested in *output per worker* and *output per capita* (that is, per person) in any nation. If we divide both sides of (5.6) by L (with $h = 1$) we get output per worker:

$$\frac{Y}{L} = y = Bk^{\alpha}, \tag{5.9}$$

where:

$$k \equiv \frac{K}{L}. \tag{5.10}$$

Output per worker in the economy depends on technology B, and the amount of capital per person (the capital-labor ratio), again assuming that $h = 1$ everywhere. Countries with lots of capital (in the form of machines) and relatively few workers will have high standards of living.

5.4 Growth Equation for k

We have already seen that the growth rate of labor is the constant n. Things are not so easy in the case of capital and output. In these cases, the growth rate is not constant over time, and depends on the state of the system.

Using our results from the last chapter, we can always express the growth rate of a ratio – like k – as the *difference* between the growth rates of the numerator and the denominator. That is, over a short span of time, any continuous variable behaves just like an exponential process. So in the case of k have:

$$g_k = g_K - n \implies \frac{\dot{k}}{k} = \frac{\dot{K}}{K} - n \ . \tag{5.11}$$

Multiply both sides of (5.11) by k then use (5.1) and (5.4) to get:

$$\dot{k} = \frac{\dot{K}}{L} - nk \ = s\frac{Y}{L} - \delta k - nk = sy - (\delta + n)k \tag{5.12}$$

Use (5.9) to re-write this as:

$$\dot{k} = sBk^{\alpha} - (\delta + n)k \ . \tag{5.13}$$

This is the *basic growth equation for the capital labor ratio*.

Figure 5.1 shows how to find the path of k through time. There are three curves, one drawn for $y = Bk^{\alpha}$, one showing $sy = sBk^{\alpha}$, and one drawn for the straight line $(\delta + n)k$. To understand Figure 5.1, note that for any k the quantity sBk^{α} is either larger or smaller than the quantity $(\delta + n)\,k$. If larger, then k itself is *rising*; if smaller, then k is *falling*.

The long-run, balanced-growth equilibrium point is determined by Point Q. Let us call this value k^*. At Q, the $(\delta + n)\,k$ curve intersects the sBk^{α} curve, so that net saving of new capital just balances the depreciation of existing capital, plus provides enough capital to equip all the new people being born.

For values of k *below* k^*, saving is greater than depreciation and population growth, so k is *growing*. For values of k greater than k^*, saving is not enough to replenish the stock and provide for new workers: k is *falling*. In this sense, the system is stable.

To find the balanced-growth value of k, set $\dot{k} = 0$ in (5.13) and solve for k. This gives us k^*, which is called the *steady-state stock of capital*, *per worker* since if $k = k^*$ then the capital stock is not growing nor is output per capita y changing, as we may see from (5.9).

Graphically, we find k^* where the lower two lines in Figure 5.1 cross. *The important point to understand is that k rises whenever it is below k*.* If k is above *k**, it falls. In other words, k is *attracted* to the unique level *k** and it will approach that value over time.

5.5 The Steady State

The value of k goes to its steady-state value over time. To find this value, as noted above, we have to set (5.13) to zero, and solve for k. This procedure yields the following expression for the steady-state *k:*

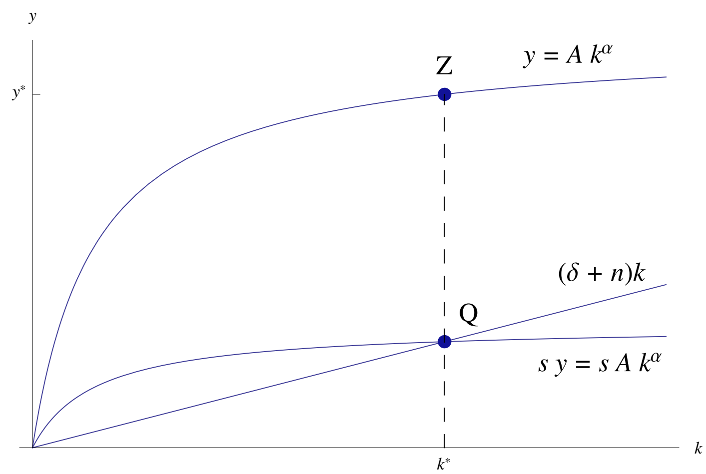

Figure 5.1: Steady State in the Neoclassical Model

$$k^* = \left(\frac{sB}{\delta+n}\right)^{\frac{1}{1-\alpha}} = A\left(\frac{s}{\delta+n}\right)^{\frac{1}{1-\alpha}} = 598.49 \tag{5.14}$$

Now substitute (5.14) into (5.9) to find the value of per-worker output in the steady state:

$$y^* = B * (k^*)^\alpha \quad = A^{2-\alpha}\left(\frac{s}{\delta+n}\right)^{\frac{\alpha}{1-\alpha}} = 319.20 \tag{5.15}$$

Consumption per worker can be found from the above:

$$c^* = (1-s)y^* \tag{5.16}$$

We can use the above equations to do simple policy experiments concerning the effects of changes in saving rates, technology, and population growth. As we see from Equations (5.14) and (5.15), increases in technology B or decreases in population growth n are always good: they raise k^* and y^* and therefore, c^* in Equation (5.16).

The saving rate s presents a different case. An increase in s raises y^*, but reduces $(1-s)$, so there are two competing effects on the consumption of each worker c^* – see (5.16). In fact, there is a unique value for s such that

c^* is maximized. This is the so-called *Golden-Rule saving rate.* Edmund Phelps won the Nobel prize in 2006 for discovering this rate. It would never be prudent to save more than the Golden-Rule saving rate. In fact, even that rate is too large to be optimal since it ignores the fact that present sacrifice is worth more than future steady-state consumption.

Finally, notice that by (5.11) in the steady state the stock of capital K grows at the same rate as the labor force L; that is, at the rate n. If this were not true then k could not be constant, as it is in the steady state. In other words, over time the growth in K adjusts *down* to n, until the ratio k is constant. Growth falls because as k rises the marginal productivity of capital declines, reducing the growth rate of K to the rate n.

Growth ceases in the Neoclassical model. In the absence of technological change, the economy's living standard approaches a constant. We know that this cannot reflect the actual course of history. Thus, if this model is correct, technical change – a rise in A – must have played a major role in lifting the standard of living in the last two centuries. The growth that comes from accumulating capital is at most *transitional* growth, growth that may be important in certain phases, but ultimately limited.

5.6 The Growth Rate of k

The capital-labor ratio will only be constant in the steady state. In general, the following expression – derived from (5.13) – gives the *growth rate* of k when the economy is not at the steady state:

$$g_k = \frac{\dot{k}}{k} = s\left(\frac{B}{k^{1-\alpha}}\right) - (\delta + n) = s\left(\frac{A}{k}\right)^{1-\alpha} - (\delta + n) \tag{5.17}$$

This growth rate is illustrated as a function of k in Figure 5.2. The growth rate is zero – so the steady state is attained – where the line crosses the horizontal axis. This occurs at k^*, which is the same capital per worker as in Figure 5.1. The growth rate is positive, but falling, whenever the actual value of k is less than k^*; it is negative, but rising, when $k > k^*$. This is what we mean by *convergence*.

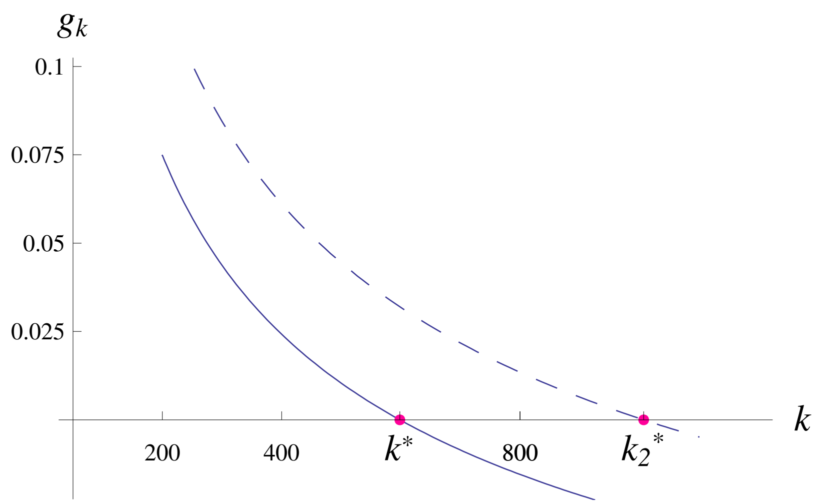

Figure 5.2: The Growth Rate of k

The existence of convergence, or the lack of it, is a topic of active investigation. If each nation had the same s, A, and n, they would all converge to the same income level. That is, y^* and k^* would be the same across countries if these three exogenous variables were the same. This is what is call *Absolute Convergence*. We do not observe this in the data, which suggests that there are differences in at least one of the three variables (or that the model is wrong – a possibility we consider later).

Figure 5.2 shows a case of *Conditional Convergence*. Think of the dashed line as pertaining to a country with better technology or laws, which makes productivity A higher than in the other country. The rich country will converge to k_2^* while the country with the low A will converge to k^*. Notice that in this case the growth rate of k will be higher if either the beginning level of the capital stock k_0 is low, or if A (or s) is high. We will test for conditional convergence using real data later.

5.7 Continuous Technical Change

We now return to (5.1) and (5.6) and assume that A – not B – is rising continuously at the rate g. That is: $A(t) = A\,(0)\,e^{g*t}$. This means that B is rising at the rate $(1-\alpha)\,g$.

From (5.1) we see that:

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta\ . \tag{5.18}$$

Let us assume that a balanced-growth path exists along which K is growing at a constant rate. If so, the above shows that $\frac{Y}{K}$ is constant, so that Y grows at the same rate as K: $g_Y = g_K$. Now take the growth rate of the production function (5.6) to get:

$$g_Y = (1-\alpha)\,g + \alpha g_K + (1-\alpha)n\ . \tag{5.19}$$

This uses the fact that $g_B = (1-\alpha)\,g$.

Substitute in $g_Y = g_K$ – since in the steady state they are equal – and recognize that $g_y = g_Y - n$ to see that:

$$g_y = g_k = g\ . \tag{5.20}$$

So the rate of technical progress determines the growth rate of per capita income y and capital per worker k. Given (5.16), we also know that in balanced growth $g_c = g_y = g_k = g$.

5.8 Endogenous Growth: A Simple Alternative Model

Assume that total output Y depends only on a broad interpretation of K, one that includes human capital. Labor does not even enter the production function. So total output is given by:

$$Y = AK\ . \tag{5.21}$$

In this simple formulation, there are no diminishing returns to capital. In essence, $\alpha = 1$, so the g_k curve in Figure 5.2 becomes horizontal, so the growth rate is

$$g_k = sA - (\delta + n) \tag{5.22}$$

.

This is the simplest in a class of models for which positive growth shows no signs of diminishing, no matter how great the stock of capital becomes. One important implication is that *there is no cross-country convergence*, either absolute or conditional. Countries that are similar to each other (in terms of s, A, δ, and n) whose y is below that of leaders never catch up. A second implication, however, is that if a nation gets better technology A (or saves more, s) it will *grow faster forever*, not just temporarily.

5.9 Growth Accounting

Is there any way to tell how much of observed growth in a country is due to changes in inputs (K and L) and how much is due to technical change A? The answer is Yes, but only if we assume that the economy is competitive. Take the production function (5.6) and express it in growth terms as:

$$g_Y = g_B + \alpha g_K + (1-\alpha)(g_h + n). \tag{5.23}$$

The basic insight, due to Solow (1957), is that α is equal to *capital's share of national income* if factor markets are competitive. With perfect competition, the payment to capital owners is equal to the marginal product of capital (see the appendix). When this holds, it can be shown that a constant fraction of GDP is paid out to owners of capital, and that fraction just happens to equal α. We can estimate α, then, by using real-world data to find the fraction of GDP that is paid out to capital owners.

We have data on everything in (5.23) *except* $g_B = (1-\alpha)g$, so we can find the growth in technology as the *residual*. That is, solve (5.23) for g_B to get:

$$g_B = g_Y - \alpha g_K - (1-\alpha)(g_h + n). \tag{5.24}$$

This equation has served as the basis for countless empirical studies of technical change, both cross-section and time series. Often, the relationship is expressed in *intensive form:*

$$g_B = g_y - \alpha g_k - (1 - \alpha) g_h. \tag{5.25}$$

This expression shows technological growth in terms of *per worker* output and capital growth. Solow (1957) was the first to try to quantify how much of the change in y was due to A. He estimated the growth in B (which is $(1 - \alpha)\, g_A$) for the United States over time and found a result which we see again and again: a surprising amount of the increase in y cannot be explained by changes in k (Solow assumed that $g_h = 0$) . It appears to be due to changes in A. Such results shift the focus of our inquiry to the processes that generate A.

5.10 Conclusion

The neoclassical model has proven to be an extremely valuable tool for thinking about growth. It has formed the basis for hundreds of theoretical and empirical studies, and has enabled policy-makers to act constructively with respect to issues of practical importance. Its shortcomings have pointed the way to a fuller understanding of the fundamental issues that must be addressed.

Appendix to Chapter 5

The marginal product of capital – or MPK – is defined to be the increase in y per unit of increase in k:

$$\text{MPK} = \frac{\Delta y}{\Delta k} \tag{5.26}$$

The MPK is a technical concept, and is derived from the production function only. It is a very important concept in economics, because it determines the rental rate on machinery and the interest rate on bonds.

The marginal product of capital is another application of the *derivative* that we have seen here and in Chapter 4. Instead of measuring the change in a variable per unit of time, it measures the increase in output per unit of capital (when the change is an infinitesimal fraction of a unit).

Linear Technology

If the production function has the simple, *linear* form $y = Ak$ (as in the model of Section 5.8 of Chapter 5) then the marginal product is just A.

The derivative MPK $= \frac{\partial y}{\partial k} = A$ follows straight from the rules of calculus and can be verified easily numerically. For example, in Equation (5.21) set $A = 10$ and $k_0 = 100$ so that $y_0 = 1000$. Now increase k by .1 units to $k_1 = 100.1$ and note that y rises by 1.0 unit. to $y_1 = 1001$.

Cobb-Douglas Technology

If the production function is Cobb-Douglass, then we use (5.6): $y = Bk^{\alpha}$. In that case, the marginal product of capital per person is given by the more complicated formula for the derivative:

$$MPK = \frac{\partial y}{\partial k} = \alpha B k^{\alpha - 1} \tag{5.27}$$

Now, if we assume that $B = 10$ and $k = 100$, and add that $\alpha = .3$, we find that $MPK = 0.119432$.

This tells us that the marginal product of capital is 0.119432 units of output, when the economy begins with 100 units of capital. That is, if you began with 100 units of capital and increased the stock by a "very small" amount, output would increase by 0.12 units *per unit of capital increased.* By how much would y increase if you raised k by one full unit (as opposed to a "very small" amount)? The answer is given by the following formula:

$$\Delta y = A(k_1^{\alpha} - k_0^{\alpha}) = 10 * (101^{.3} - 100^{.3}) = 0.119016 \tag{5.28}$$

It is close, but not the same.

Payment to Owners of Capital

If the market for renting capital is competitive, then owners can charge a rental rate equal to the marginal product of capital:

$$R = MPK = \alpha B k^{\alpha-1} \tag{5.29}$$

Notice that how much you get for your capital depends on the amount of capital per person in the economy already (k). The share of national income that goes to owners of capital is $\frac{RK}{Y}$. Making the normal substitutions yields:

$$\frac{RK}{Y} = \frac{Rk}{y} = \frac{\alpha B k^{\alpha-1} k}{B k^{\alpha}} = \alpha \tag{5.30}$$

So, given the production function, the owners of capital always received a constant share of GDP (national income). That share is the same as the α in the production function.

Chapter 6

Regression Analysis

6.1 Introduction

Regression analysis is a method that allows us to quantify the association between two variables that we observe in nature or in experimentation. It is a very common tool of research in economics, business, finance, political science, and public policy. This chapter gives a brief conceptual description of the technique, as well as a practical guide to actually performing a regression between two or more variables.

6.2 The Basic Theory Illustrated with Simple Data

Let us say that we think that nations with high levels of openness to trade and investment (the variable x) are also countries with high growth (the variable y). That is, we might hypothesize that the higher is x, the higher is y. Indeed, we might go even farther and say that we suspect that the true relationship between x and y is:

$$y_i = \alpha + \beta x_i + \epsilon_i \tag{6.1}$$

This is a *linear* relationship. Its meaning is that the growth rate of a country i (y_i) is determined in a *common way* in every country, so differences in growth arise only because of differences in the particular openness (x_i) – plus a random error term ϵ_i – for each country. The random error – whose mean is zero – should be thought of as containing the influence of other variables that we cannot observe (or have not bothered to observe).[1]

Table 6.1 gives some data (this is artificial data) on y and x for 20 different countries. Each pair of numbers represents a country for a particular time span, say, 1960-1970. This data is graphed in Figure 6.1: it is represented by the solid dots. Each one is a country. Evidently, there is a positive association between x and y. There are two questions that we would like to have answered:

1. What is the "best" straight line through the points in Figure 6.1? Knowing the answer to this question is the same as knowing the "best" estimates of α and β in Equation (6.1) above.

2. How "good" is the "best" straight line? Is it very good? Or just barely adequate? Or thoroughly inadequate?

Regression analysis can provide answers to both of these questions.

We do not need a lot of mathematics to understand conceptually what regression analysis is all about. It is relatively simple, although putting it into practice does require a lot of math – done by computers. Regression is a technique that finds the α and β that minimize the sum of the differences (after squaring them) from each point in Figure 6.1 to the straight line defined by the α and β. In other words, any (α, β) pair defines a line like that in Figure 6.1, and for any such line we could calculate all of the twenty differences between each actual point and the line. Square them and add them up. The best line is that which minimizes those squared differences.

In this course, we do not have the time to go through the mathematics of just how this is done. There are many, many computer packages that will run a regression for you, including Excel. The output you see in the

[1] In regression analysis, y is the "dependent variable" and x is the "independent variable" or "explanatory variable". Note that we are *assuming* that causality runs from x to y, but that assumption might be false.

Table 6.1: Data on Growth and Openness

Country	y Growth Rate	x Openness
Alland	0.77	0.3
Bobland	1.46	0.4
Chad	2.78	1
Doelandia	5.55	2.3
Evergreen	2.28	0.9
Floridia	7.22	3
Georgeland	2.78	1.2
Huland	4.64	2
Idia	1.97	0.8
Johnonia	5.73	2.5
Kelland	3.05	1.6
Lidia	2.53	1.3
Melland	4.91	2.1
Nyland	1.12	0.2
Orendia	1.81	0.1
Paland	2.61	0.5
Queenland	2.43	1.2
Rickland	4.25	1.7
Stemside	2.9	0.8
Twentia	4.87	2

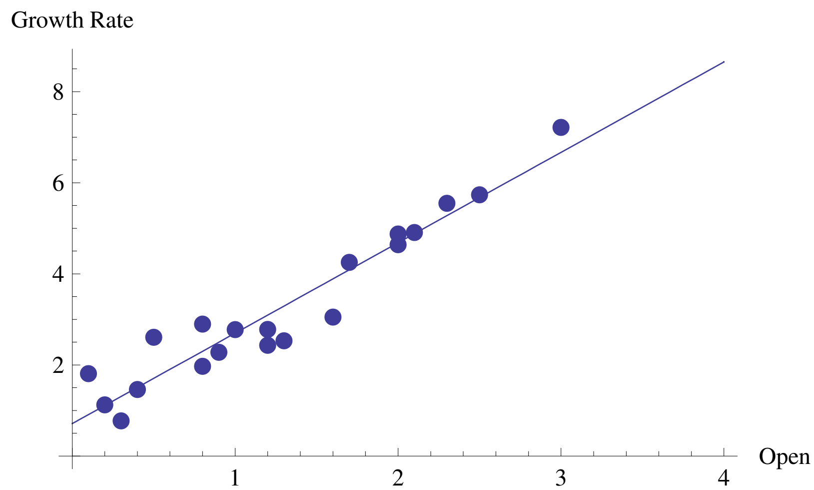

Figure 6.1: Regression Line and Actual Values

Appendix to this chapter is from a program called *R*. The regression line is shown in Figure 6.1 along with the scatter plot. The estimated equation is:

$$\hat{y} = 0.712 + 1.985x \tag{6.2}$$

The "ˆ" indicates that y defined by the straight line is an *estimated* or *predicted* value, not the actual. The $\hat{y}$ values lie exactly on the line; the actual values of y are given by the solid dots. The error term does not appear in (6.2) because the predicted error is zero.[2]

Look at the output in the Appendix. There, I ran four different, but similar, regressions. Each set of results begins with the word "Call", followed by the regression formula. After that, there are three groups of results. Look, first, at the group called "Coefficients". In the column labeled "Estimate" we see the two numbers in Equation (6.2). This tells us that if $x = 0$ (that is, a closed economy) then growth would be only 0.71 percent per year for

[2]The data in Table 6.1 was not just randomly put together to look good. It was generated by assuming the true relation was $y_i = .5 + 2x_i + \epsilon_i$ where the random error had a mean of zero and a variance of .50. The computer actually generated 20 random values, one for each country. The regression technique, notice, did not get to the truth! But it came close, especially for β (the estimate of 1.98 is very close to the true value of 2).

the average country. But, for every 1 unit increase in openness x, growth y rises by 1.985 units. Therefore, openness is very helpful for growth. The β coefficient (1.985) measures the strength of the effect running from x to y.

The other information shows us how much faith we can have in the estimated relationship in Figure 6.1.

The column labeled "t value" shows how well the individual x-variables do in explaining y. Here, we have two x-variables (we always count the Intercept or constant term), and both do well. We are looking for a t-value over 2.0, to say that the x-variable is "statistically significant" in helping to explain y. We see that we easily have that in both cases, and in the case of x (openness) it is extraordinarily significant, since 14.026 is far greater than 2.[3]

The next column, "Pr($>$ | t |)", shows the probability that the relationship between x and y is due merely to chance, not an underlying relationship as in Equation (6.1). Here, we want to see numbers below 0.05 (5%), but will often settle for numbers below 0.10 (10%). As a matter of arithmetic, whenever the t-statistic is high, the P-value is low, so we really only have to look at one of them.

The R^2, the F-statistic, and the p-value reported in the third group are a measures of the "overall goodness of fit" of the relationship. The R^2 is a number between 0 and 1, with 1 being "a perfect fit". Usually, we are content with an R^2 value anywhere above .40, or even lower in some cases, especially if we have only a few x variables. The value here of $R^2 = 0.9162$ is quite high, indicating a very good fit, as seems clear from the figure. An F-statistic over 10 is good; here it is almost 200. The p-value is virtually zero. All of these point to a great fit: the variable x explains a lot of the variation in the variable y.

We will discuss the other regression results in class.

[3]The t value is the ratio of the estimated coefficient in Column 1 and the Standard Error ("Std. Error") in Column 2.

6.3 Conclusion

Regression is a powerful technique for analyzing data, and is really not hard to put into practice. The data in this chapter was made up to illustrate how to use the technique and was constructed to deliver good results. Often, our results will not be so good.

In this simple example, by construction, there was only one variable x (openness) that influenced y (the growth rate). That is usually not true: normally, there are many variables to put on the right hand side of the regression equation. This presents no problems, but the correct specification is usually elusive, and does matter for the inferences that we are allowed to draw.

Appendix: R Output

The following is typical output from the *R* statistical package.

Chapter 7

Source of Technology

7.1 Introduction

As more attention becomes focused on "technology" A, the more important it is to think about what it is and how it is brought into existence. Our theories so far are not very far advanced. There is currently a great divide in thinking about A: does it happen by accident or is it actively sought? We take up this question in various forms throughout the chapter. The question is hard to answer because there is no easy way to define "technological progress" or to classify the different ways that new knowledge influences economic growth. The following list shows some of the concepts that might conceivably have an impact on economic growth. The list begins at the most general level and flows down to the specific.

- Science
- Ideas
- Invention
- Innovation
- Human Capital
- Skill

All of these are related to one another, and all of them are important for economic progress. Which of these are most closely related to the concept of "total factor productivity" — the parameter A — that appears in the neoclassical production function?

The closest in concept to A is probably "innovation", which is the widespread application of an "invention", which is itself the embodiment of an "idea" whose generation depended upon past "scientific discovery". Often the diffusion of an innovation requires "human capital" or "skill" in individuals.

But where does innovation come from? It stands to reason that nations with lots of human capital will innovate more than others. It is also logical that countries with better property rights will have more innovation, and perhaps more physical and human capital as well. On the other hand, nations that innovate more probably produce incentives for individuals to accumulate more human capital. We have, in short, a kind of "chicken and egg" problem.

7.2 Basic Issues and Terminology

7.2.1 Rivalry vs Non-rivalry

One of the key concepts in modern growth theory involves the distinction between "rival goods" and "non-rival goods". Most inputs in the production process are rival goods, meaning that by their very nature, if one firm uses them another firm cannot. This is true of any particular machine or worker, and thus may be identified with K and L in the neoclassical production function of Chapter 5. Inventions or innovations — in general any idea — are non-rival goods, since they can potentially be used by everyone simultaneously. A good example is a recipe, perhaps the secret formula in Coca Cola. If the bottler in Atlanta is using this invention, it in no way stops the bottler in Rio de Janeiro from using the exact same invention.

These concepts are related to the notion of diminishing returns in the production process. Usually, rival goods show diminishing returns to increases in their amounts while non-rival goods do not. Non-rival goods are at the heart of endogenous growth theory, especially the work of Paul Romer (1986, 1987, 1990). This theory shifts the focus from the accumulation of physical capital K to the generation of new ideas A. Potentially, this is a huge shift, since the lack of diminishing returns opens the possibility of growth without bound (see Section 6 of Chapter 5). Yet, until we settle on a process for generating A we are not much farther along.

7.2.2 Excludability

Just because an invention could be used by many producers at once does not mean it will be. If a firm can keep others from using its ideas, we say that the innovation is *excludable*.

There are two ways that firms exclude others: secrecy and patent law. Both of these are only imperfect mechanisms. The cotton gin is a good example of how both can fail. Eli Whitney's machine was easily "reversed engineered" so it could not be kept secret. On the other hand, all of his efforts to secure a patent came to nothing, since many of the gins were slightly different, and it was not in the interest of southern agriculture to restrict the invention. As a result, it appears that the courts were reluctant to grant him the patent, and did not do so for many years.

Even when patents work, they may not be the most efficient way to produce new ideas. There are several reasons for this. Among them: the social return to many inventions is far higher than the private return to a monopoly; others spend wasteful amounts in finding close alternatives, in order to get high profits, rather than trying to find more revolutionary new goods; patent-holders can block promising new research in allied fields; there are better ways to finance research than "taxing" the good that gets invented.

Nevertheless, it is difficult to think of a system that is clearly better than the patent system. One alternative is the purchase of the patent for the new idea by the government, who would then put it in the public domain. The shortcomings of the patent system, and a possible improvement are discussed in Michael Kremer (1998).

The reason that excludability of non-rival goods is important is because it matters greatly for the profit potential of a new idea. Once you have the idea, if you can exclude others you become a monopolist. Profits follow. And if profits exist, then people will spend time, effort, and resources looking for ideas in the first place. This is quite relevant for the drug and software industries, in which production costs for new drugs and programs are extremely small, but up-front costs of developing them are enormous. Only with some excludability will companies invest those resources. The whole concept of R&D (research and development) relies on the ability to recoup profits later. Excludability is the key to this process.

The combination of non-rivalry and excludability is at the center of those theories that propose that A is the result of deliberate search. Patents, research, and inventions represent the parts of the production function for technology. There is, however, another view.

7.2.3 Technology by Accident: Externalities, Spillovers, and Learning by Doing

At times inventions arise by accident. This may happen in one of two ways.

First, when one firm does research in a particular area, other firms may benefit indirectly. Scientists often discuss their work with others in different companies; results are presented at academic seminars; new goods are observed at various stages of development. The technology that one firm receives that it did not pay for (either through its own R&D or licensing) is called a technological *externality* or a technological *spillover*. Spillovers are unpriced, non-rival goods that arrive free to producers. By their very nature, spillovers and externalities are hard to measure.

The second accidental way that ideas arrive is through learning-by-doing. In the course of producing a good, firms simply get better at it. They find new ways of doing things that reduce costs and improve the product. Again, these innovations are free, in the sense that no one spent time or money looking for them. The first rigorous treatment of this idea was by Kenneth Arrow (1962).

Paul Romer analyzed the case of externalities and growth (Romer, 1986). His idea is that the capital investment by a small firm raised its own stock of productive machinery k, but also increased the general knowledge of the economy $K = \eta k$. where $\eta > 1$ is the number of firms. The small firm does not recognize the effect its investment has on the economy's knowledge stock, so the social return to k exceeds the private return to k.

This sort of model has two implications. First, firms do not perceive the global external effect, so they invest too little in terms of society's welfare. Second, it is possible that there are social constant (or even increasing) returns to capital. If so, the economy may never settle down to a steady state, even if there is no technological progress in addition to the knowledge created as a by-product from investment. That is, endogenous growth may arrive by accident.

The basic idea is illustrated in Figure 7.1, which shows both the individual firm's perceived (or private) production function, and the actual (or social) production function, assuming that there are increasing returns to scale when the effect of K is counted. The social function rises at an increasing rate; that perceived by each firm shows falling gains in output to increasing capital.

7.2.4 Collective Choice and Public Goods

If externalities are important, then a case can be made for collective action, in the form of provision of technology by the government (public provision). One possibility was mentioned above: the purchase of patents by the government for public distribution free of charge.

Another possible way to achieve this is to fund research. This is already done in many countries, including the US, which has several agencies (NSF, NIH, NEA, etc) responsible for giving out research funds.

The existence of externalities has led some economists to advocate the use of the government to coordinate investment plans in general, or to provide subsidies for particular "winner" industries. Only the government, according to this argument, can simultaneously take into account all of the

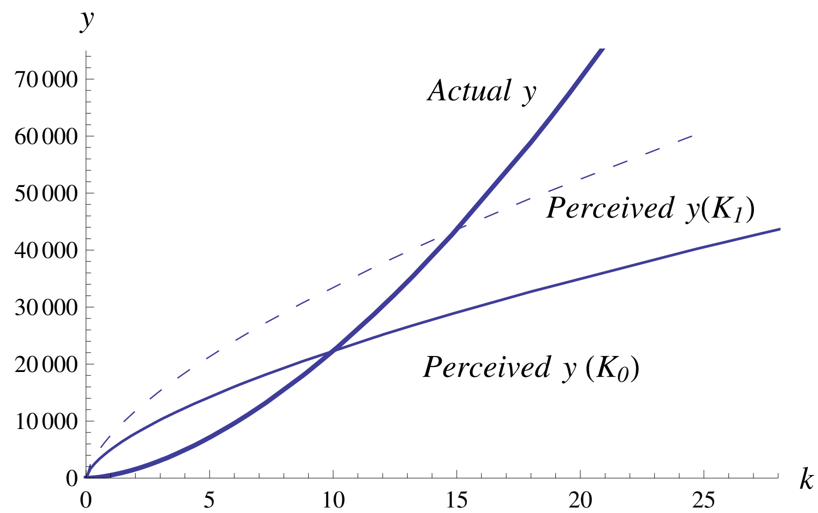

Figure 7.1: A Positive Knowledge Externality

many external benefits that exist across the millions of firms. The problems with this approach are, however, severe. The government may not have the correct information, or the ability to marshal and administer resources on a grand scale. Perhaps most importantly, the government may have objectives that conflict with those of the private sector, like war or corruption. Collective action was tried in developing economies in many parts of the world in the 50s and 60s as part of the strategy of import substitution. Henry Bruton (1998) has described this approach in detail.

7.2.5 Institutions and Technology

It is impossible to overlook the fact that society's institutions are an important influence on the pace of change. Those societies with institutions that favor and encourage innovation will achieve faster growth and higher living standards compared to others. The theme of institutional arrangements and growth is taken up by William Baumol (1990) and Kevin Murphy, Andrei Shleifer, and Robert Vishny (1991). Hall and Jones (1999) show that there is strong evidence to support the idea that institutions are the key to modern growth.

Protecting property rights and allowing profit are characteristics of economically successful societies. It is interesting that many continue to violate both. There is a large and growing literature on the political obstacles to liberalization, although not a lot of it has been related to growth (McDermott, 1997, 1998)

Religion may be important. Max Weber proposed that Protestantism was the key to capitalist development. Others have pointed out that the causality may have run the other way.

7.3 Technology and Evolution

Evolutionary processes have been proposed for many biological and social organisms. Joel Mokyr (1990) uses evolution to discuss the progress of inventions through time.

Ideas evolve in leaps. Large ideas occur infrequently, and often by accident — the macro-inventions. Smaller ideas that improve the large important ones – the micro-inventions – occur much more routinely and as a function of economic incentives. Therefore, technical change is continuous, but its rate of increase is not: once in a while there will be a large jump in productivity.

The world economy, as a result, may be characterized by "punctuated equilibrium", to borrow a phrase from evolutionary biology.

7.4 Historical Approaches to Technical Progress

Nathan Rosenberg (1982) has studied the concept of technical change as it has changed through history.

7.4.1 Types of Innovation

First, there are two types: product innovation and process innovation.

Simon Kuznets and Joseph Schumpeter emphasized the former over the latter, but most theoretical work has dealt with the latter since it is easier.

Both also focused more on discontinuity and the jumps coming from large, key inventions of new goods.

7.4.2 Continuity

Rosenberg emphasizes that there were several economic historians who thought that a slow, continuous process was a better way of characterizing the history of technology. These include Marx, A. P. Usher, and S. Gilfillan.

7.4.3 Direction of Innovation: Labor or Capital?

Did inventions in America tend to be labor-saving because labor was scarce in the New World? This has been held true by several writers, but others dispute it. This controversy has been re-opened by Romer (1996) who looks at the relative growth of the US in the early part of our history.

7.4.4 Diffusion of Innovation

How do inventions diffuse through the world? Are they fast or slow? Again, it depends on institutions, and perhaps the movement of the people themselves. There is a sizeable literature on the economics of imitation and growth in developing countries.

7.5 Conclusion

This chapter has set forth the basic concepts of technological change and growth. Policies, institutions, and external events are important to the growth process only insofar as they affect the rate of technological change. Technology stands behind every one of the various processes that we examine.

Chapter 8

Perpetual Growth and Finite Resources

In this chapter, we present a model of growth with natural resources. It turns out that steady growth is possible if technological change is sufficiently strong. The production of light is a famous example of the power of technology to overcome resource scarcity: the use of resources in producing lumens has fallen tremendously as technology has allowed us to substitute wood, for candles, for whale oil, to petroleum, to incandescent, to fluorescent, to LEDs.

8.1 Extraction and Growth

The production function is very simple:

$$Y = AR^{\beta} \tag{8.1}$$

where A is technology and R is the *flow* of resources. This is the amount of newly produced resources every time period. Everything is produced by resources only, with diminishing returns, but enhanced by technology. Per capita output is:

$$y = \frac{AR^{\beta}}{N} \tag{8.2}$$

Let S be the stock of natural resources so:

$$R = -\dot{S} = -\frac{dS}{dt} \tag{8.3}$$

Certainly, if R is *constant, this cannot go on forever.* It would not be sustainable. So let's adopt a "Zeno's paradox" extraction rule:

$$R\left(t\right) = \lambda S\left(t\right) \tag{8.4}$$

where λ is a constant rate. This rule says: "Take a constant *fraction* of what is left of the stock at any moment". It may be easier if you think of t as a year. So, in 2011 we would take, say, 2%, of the stock of resources that was in existence at the beginning of 2011. Then, in 2012, we would take another 2% of the stock that existed at the beginning of 2012. So, if we started 2011 with a stock of size S_0, at the end of 2012 we would have $S_2 = \left(1-\lambda\right)\left(1-\lambda\right)S_0$.

It may seem paradoxical, but for *any constant* λ, you never run out of the natural resource, even if $\lambda = .80$. If, for example, this went on for any length of time T, we would still have $S_T = \left(1-\lambda\right)^T S_0$ units of the resource left. This number is always positive.

Although discrete time is usually easier to think about, continuous time yields simpler results. So now we switch to continuous time.

Combine (8.3) and (8.4) to see that:

$$\frac{\dot{S}}{S} = -\lambda \tag{8.5}$$

The stock S falls at the rate λ always. Now rearrange (8.4) to see that:

$$\frac{R}{S} = \lambda \tag{8.6}$$

Since λ is a constant, that means that R and S change at the same rate. Since S is falling at the rate λ, then we know R is also falling at that rate:

$$g_R = \frac{\dot{R}}{R} = -\lambda < 0 \tag{8.7}$$

This just says we take out a smaller and smaller amount of resources every year. Both the stock and the flow decline steadily.

The Appendix to this chapter gives us another way to think about the extraction rule.

"Growth" means that the growth rate (i.e. rate of change) of y is positive. From (8.2), we find the growth rate of y to be:

$$\begin{aligned} g &= g_A + \beta g_R - g_N \\ &= g_A - \beta\lambda - g_N \end{aligned}$$

For g to be positive forever, we need $g_A > \beta\lambda + g_N$. This does not seem that difficult to achieve, especially if population growth is low. For example, if $\beta = .5$, technical change is 4% a year, and population growth is 1.5% per year, we need to extract our resources at any rate less than 5% to maintain growth in living standards forever. That is, $\lambda < .05$ guarantees positive growth.

To summarize, any value of λ is sustainable – S will never run out – but not every value of λ is capable of maintaining growth in y forever.

8.2 Optimal λ

Do we need growth? How do we think of the "best" or "optimal" rate of extraction?

One way to consider optimality is to add up all the current and future output of the country (or the world). Usually, we discount future output (which equals income) such that the farther away it is in the future, the less valuable it is. This is very similar to the idea behind present value of a future asset, but is different in that it contains an implicit valuation of the utility of generations that are not yet born.

In discrete time, we would maximize:

$$J_D = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t y_t$$

where ρ is the subjective rate of time preference. In continuous time we would maximize:

$$J_C = \int_0^\infty y(t)\, e^{-\rho t} dt \tag{8.8}$$

From the first section, we can write

$$y(t) = y(0)\, e^{gt}$$

where $g \equiv g_A - \beta\lambda - g_N$. Putting this into (8.8) gives us:

$$J_C = \int_0^\infty y(0)\, e^{-xt} dt = y(0) \int_0^\infty e^{-xt} dt$$

where $x \equiv \rho - g = \rho - g_A + \beta\lambda + g_N$. We can take the term $y(0)$ out from under the integral sign because it is a constant. It is necessary that $x > 0$ so that the integral converges.

We can express initial output as:

$$y(0) = A_0 (\lambda S_0)^\beta$$

because our rule says that $R_0 = \lambda S_0$. The integral can be solved to get:

$$\int_0^\infty e^{-xt} dt = \frac{1}{x}$$

So that:

$$J_C(\lambda) = \frac{A_0 (\lambda S_0)^\beta}{x} = \frac{A_0 (\lambda S_0)^\beta}{\rho - g_A + \beta\lambda + g_N}$$

What value of λ maximizes this expression? It is not at all clear from looking at it. We can see, however, that an increase in λ is ambiguous without more information about the parameter values, since it raises both the numerator and the denominator.

We can use calculus to find the optimal value. Or, we can plug in numbers for the parameters other than λ and draw a graph. In **Figure** 8.1 we show the graph of J_C as a function of λ. We assume that $A_0 = 10$ and $S_0 = 1000$. There is no population growth so $g_N = 0$. We also assume that

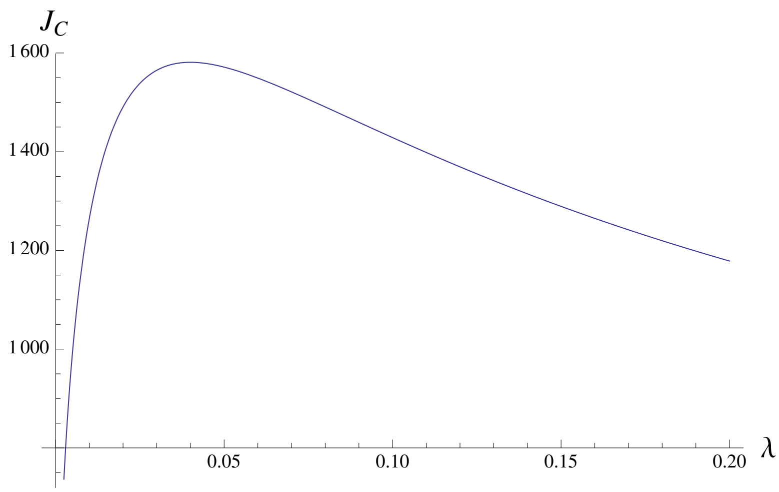

Figure 8.1: The Optimal λ

$\rho = .04$, $g_A = .02$, and $\beta = .50$. It looks like the best value is around 4%.

In fact, the computer confirms that the government should set $\lambda^* = .04$: extraction should proceed at 4% of the available stock. In the first year, the economy should extract 40 of the available 1000 units (S_0). Notice that this solution gives zero growth: $g = 0$. The best that we can do is consume the same amount every year.

What if people did not discount the future so much? If we reduce the rate of time preference to $\rho = .03$, then the optimum falls to $\lambda^* = .02$. Growth, in this case, is positive forever: $g = .01$.

Appendix: Extraction Rule Revisited

Let us consider extraction from a slightly different perspective. Assume the government or business decides to begin extracting at the rate $R(0)$ and has committed to reducing the extraction flow continuously at the rate γ. Both the initial value $R(0)$ and γ are independent at this point. This means that $R(t) = R(0)\, e^{-\gamma t}$. How much resource would be extracted by the end of time (infinity)? Call that amount S_∞. It is:

$$S_\infty = \int_0^\infty R(t)\,dt = \int_0^\infty R(0)\,e^{\gamma t}dt \tag{8.9}$$

Now, solve the integral to get:

$$S_\infty = \frac{R(0)}{\gamma} \tag{8.10}$$

So the greater the initial rate of extraction $R(0)$ or the smaller the rate of decline in the rate γ, the more will be extracted in total.

The problem is that the world may not have an amount as big as S_∞. If we want to extract exactly the amount we have S_0 in the limit of time, then we must set $S_0 = S_\infty$ and solve (8.10) to get:

$$R(0) = \gamma S_0 \tag{8.11}$$

But this is precisely the rule we have proposed, except that we called the rate λ before (not γ).

For any country the stock S_0 is given (but may be uncertain) and either γ or R_0 may be chosen independently, subject to $0 < \gamma < 1$. The other is then determined by (8.11). After that, the country must commit to reducing R each period according to the commitment $R(t) = R(0)\,e^{-\gamma t}$. If they do, then the resource will last forever.

Chapter 9

The Limits to Wage Equalization Across Countries

9.1 Introduction

Wages are very different across countries. It is important to think about why these differentials arise and what we can do about them. Among the most important questions may be this: has globalization reduced the wage differential or increased it?

It turns out that we can provide answers to this question and others using the basic neoclassical production function and the assumption that markets for factors (capital and labor) are competitive.

9.2 Comparing Separate Economies

At first, let us consider two economies that are completely separate. This means that we can analyze relative income, wages, and returns to capital in each country independently. Later, we allow globalization in the form of movement of capital. We will then see how this affects wages and returns to capital in the two countries.

9.2.1 Production and Relative Per Capita Income

We begin with the production functions in two countries, an industrial country (the US, for example) and a developing country (perhaps India). Let us write them as follows:

$$Y_N = A_N K_N^{\alpha} (h_N L_N)^{1-\alpha} \tag{9.1}$$

$$Y_S = A_S K_S^{\alpha} (h_S L_S)^{1-\alpha} \tag{9.2}$$

The subscript "N" stands for the industrial north and the subscript "S" stands for the southern LDC. As usual, we have the following interpretations of each variable: A is technology (broadly interpreted), K is capital, L is labor (number or hours), and h is the skill (education) level of each person.

Define the aggregate human capital stock as:

$$H \equiv hL \;\Rightarrow\; h = \frac{H}{L} \tag{9.3}$$

and the capital-labor ratio as usual: $k \equiv \frac{K}{L}$. As always, our basic welfare criterion is output per person, $y \equiv \frac{Y}{L}$. For the two countries y can be written using (8.4) as follows:

$$y_N = A_N k_N^{\alpha} h_N^{1-\alpha} \tag{9.4}$$

$$y_S = A_S k_S^{\alpha} h_S^{1-\alpha} \tag{9.5}$$

I will call the relative per capita income in the two countries:

$$z_y = \frac{y_N}{y_S} \tag{9.6}$$

We now derive expressions for the returns to capital and labor.

9.2.2 Return to Capital

The return to capital r – the rental price that a firm would pay for a machine – is the same as the marginal product of capital. As we noted in Chapter 5, the marginal product of capital is the derivative of Y with respect to K.

For either country j, it is given by:

$$r_j = \alpha A_j \left(\frac{h_j}{k_j}\right)^{1-\alpha} \quad j = (N, S) \tag{9.7}$$

Countries with more human capital h per machine k have higher returns to physical capital k . It certainly makes sense that your machinery (that is, capital) is more productive when your workers are more highly educated.

9.2.3 Return to Labor

The return to labor is the real wage – the rental price for worker services. In a model with human capital we must distinguish between the wage of a worker w_L and the wage of a unit of human capital, w. For an individual worker they are related as follows:

$$w = w_b * h \tag{9.8}$$

Equation (9.8) says that a worker earns a base wage w_b (the same as the wage of a unit of human capital) times the amount of human capital she possesses. Workers in all countries have different levels of human capital, determined by their education, experience, and ability. A worker with human capital level x makes x times more than a worker with only 1 unit of human capital. This is true of both countries, so we can write:

$$w_j = w_{b,j} h_j \qquad j = (N, S) \tag{9.9}$$

where h_j is the human capital of an *average worker* in Country j for $j = (N, S)$. The question is: how do we find the equilibrium values of the base wages $w_{b,N}$ and $w_{b,S}$?

The base wage w is the marginal product of H (the derivative of Y with respect to H). Using the production functions (9.1) and (9.2) and the definition (9.3) these give:

$$w_j = (1-\alpha)A_j \left(\frac{k_j}{h_j}\right)^{\alpha} \qquad j = (N, S) \tag{9.10}$$

There are two profound implications of the above equations. The first is that the more capital (machines, inventories, etc.) a nation has, the *higher will be its wage*. The second is that the more human capital (that is, skilled workers) a nation has, the *lower will be the wage for any particular skill level*. This does not mean that education is bad. If everyone gets more skill or education, wages will rise. How can that be true? Look at (9.8). If everyone's h rose, w would fall, but wh would rise. That is, w falls by α percent (where $\alpha < 1$) of the rise in h.

The ratio of the base wage in the industrial country to that in the LDC can be found from (9.10):

$$\frac{w_N}{w_S} = \frac{A_N}{A_S}\left(\frac{k_N h_S}{k_S h_N}\right)^{\alpha} \tag{9.11}$$

Use (9.4), (9.5), and (9.6) to express it compactly as:

$$\frac{w_N}{w_S} = z_y \frac{h_S}{h_N} \tag{9.12}$$

It is important to emphasize that the ratio above is for *base wages*. For actual wages, use (9.12) along with (9.9) to see that:

$$\frac{w_{L,N}}{w_{L,S}} = \frac{w_N h_N}{w_S h_S} = z_y \tag{9.13}$$

This result is true for *all* of our cases in this section that deals with two separate economies. The ratio of actual wages in the two countries is the same as the ratio of per capita incomes.

9.2.4 Examples and Cases

Our purpose in the rest of this section is to calibrate the values of A, K, and h in each country to see what we can infer about relative income, wages, and the return to capital when the countries are separate. For our case, we take the US for the industrial country and India for the LDC (a popular comparison). Our main fact is from the Summers-Heston-Aten data set. There we see that in 2000 the US y was 33,308 I\$ whereas India's y was only 2,480 I\$. Thus, the ratio of the two was 13.43. For simplicity of exposition, we round this up and set

$$z_y = 14$$

Counting all kinds of income, US residents were about 14 times better off than their Indian counterparts.

9.2.4.1 Case 1: A = 1 and h=1 in Both Countries

When the neoclassical growth model was first being developed and applied, it was customary to ignore h (that is, set $h_n = h_S = 1$) and assume that technology was the same all over the world ($A_N = A_S = 1$). With those two assumptions and assuming that $\alpha = \frac{1}{3}$ (again, a standard number) we use (9.4) – (9.6) to write:

$$z_y = 14 = \left(\frac{k_N}{k_S}\right)^{\frac{1}{3}} \tag{9.14}$$

Solve for k_N to get:

$$k_N = 14^3 k_S = 2,744 k_S \tag{9.15}$$

Can the US really have 2,744 times more capital per person than India? This would appear to be way too high. But let's suspend our disbelief for a second and ask: if this were indeed true, what does it imply about wages and the return to capital? Use (9.7), and our assumed values $A_N = A_S = h_N = h_S$, to get:

$$\frac{r_S}{r_N} = \left(\frac{k_N}{k_S}\right)^{\frac{2}{3}} = \left(14^3\right)^{\frac{2}{3}} = 196 \tag{9.16}$$

This says that the return to capital should be about 196 times greater in India than the US! Again, this seems unreasonable large. If this were true, why doesn't capital rush to India? Why settle for such a low return in the US and Europe?

Since $h_N = h_S = 1$, in this case, base wages and actual wages would be the same. Moreover, they would have the same relative value as income:

$$\frac{w_N}{w_S} = z_y = 14 \tag{9.17}$$

This result comes straight from (9.12) and (9.13) and means that a worker who lived in India but had the same human capital as a US worker could increase her wage 14-fold if she moved to the US. That may explain why we see so many people of all education levels trying to move to the US and Europe.

9.2.4.2 Case 2: More Human Capital in the Industrial Country

We know that the typical worker in the US has more human capital than her counterpart in India. Robert Barro and Jong-Wha Lee have estimated that in 2000 the typical worker in the US had 12.25 years of schooling, compared to 4.77 years for a worker in India. (See Robert Barro's website at Harvard to get this data for many countries and years.) We may infer from this that average human capital is about 2.57 times greater in the US than India. Let us round down and set:

$$\frac{h_N}{h_S} = 2.5 \tag{9.18}$$

Now the capital stock in the US need not be so huge to generate the 14-fold per capita income gap. That is, a part of the income gap can be explained by differences in human, not physical, capital. Using (9.6) and (9.18), but still assuming $A_N = A_S = 1$ we now find:

$$\frac{k_N}{k_S} = (z_y)^3 \left(\frac{h_S}{h_N}\right)^2 = \frac{14^3}{2.5^2} = 439 \tag{9.19}$$

This is more reasonable, but a 439 multiple is still vary large.

The return to capital in India would be still quite a bit larger than in the US. Begin with (9.7) then use (9.18) and (9.19) to eliminate the ratios $\frac{h_N}{h_S}$ and $\frac{k_N}{k_S}$ to get:

$$\frac{r_S}{r_N} = \left(\frac{h_S k_N}{h_N k_S}\right)^{1-\alpha} = \left(\frac{439}{2.5}\right)^{\frac{2}{3}} = 31.41 \tag{9.20}$$

Now the return to capital is only about 31 times greater in India than in the US.

As for base wages, the gap is also smaller. Using (9.12), we get:

$$\frac{w_N}{w_S} = z_y \frac{h_S}{h_N} = \frac{14}{2.5} = 5.6 \tag{9.21}$$

We must emphasize that these are *base wages*; that is, the wage for a person with one unit of human capital. The ratio of average wages is still 14 ($w_L = wh$) since the average US worker has 2.5 times the human capital of her Indian counterpart. Consider a doctor, for example, whose $h = 22$, say. In the US, a doctor would make about \$100,000, let's say. According to (9.21), the same doctor in India would make \$100,000/5.6 = \$17,857, assuming they had the same human capital. That is, for comparing people with the *same* human capital, we use the ratio of base wages. To compare countries, where the average human capital can be quite different, we use (9.13) and compare *actual average* wages.

As Easterly emphasizes several times in his book *The Elusive Quest for Growth*, since the US has over twice as much human capital as India (per worker) the price of a unit of human capital (the base wage) should be correspondingly *smaller* in the US – at least if we maintain the strict neoclassical assumption of diminishing returns to H. That may not be reasonable, as Easterly himself constantly emphasizes. The base wage gap is still positive in favor of the US – and sizeable – even after we account for human capital. This is so because there is considerably more *physical capital* per person k in the US, too, which raises the productivity of human capital and makes educated workers at all levels 5.6 times more productive than in India.

9.2.4.3 Case 3: A and h Greater in the Industrial Country

We now come to our last case involving two *isolated* economies. It may seem unreasonable to assume that in India and the US technology is the same: $A_N = A_S$. We may instead believe that technology (TFP or total factor productivity) is better in the US, especially when we interpret it broadly to include property rights, justice, financial markets, security, and other relevant institutions. Accordingly, in this section, we assume that:

$$\frac{A_N}{A_S} = 2.0$$

Now, we use (9.4) – (9.6) to arrive at the following general formulation:

$$\begin{aligned}\frac{k_N}{k_S} &= (z_y)^{\frac{1}{\alpha}} \left(\frac{A_S}{A_N}\right)^{\frac{1}{\alpha}} \left(\frac{h_S}{h_N}\right)^{\frac{1-\alpha}{\alpha}} \\ &= \left(\frac{14}{2}\right)^3 \left(\frac{1}{2.5}\right)^2 = \ 54.88 \qquad (9.22)\end{aligned}$$

So now we have the relative capital-labor ratio down to 55 times larger in the US. It is difficult to know if that is, in fact, reasonable, but it seems more realistic than our earlier results.

Using (9.7) we may express the relative return to capital as:

$$\frac{r_S}{r_N} = \frac{A_S}{A_N} \left(\frac{h_S}{h_N} \frac{k_N}{k_S}\right)^{1-\alpha} = 3.92 \qquad (9.23)$$

We know all of the ratios in (9.23), including that for k from (9.22), and if we plug them in we get the answer noted on the right above, 3.92. This is much smaller than our previous results and seems more reasonable. The return to capital is higher in India, but only by a factor of 4 now.

Now use (9.22) in (9.23) to get the following expression:

$$\frac{r_S}{r_N} = (z_y)^{\frac{1-\alpha}{\alpha}} \left(\frac{A_S}{A_N}\right)^{\frac{1}{\alpha}} \left(\frac{h_S}{h_N}\right)^{\frac{1-\alpha}{\alpha}} = 3.92 \qquad (9.24)$$

This has the advantage of involving only quantities for which we have data or educated guesses. Equation (9.24) is the expression that corresponds to (9.12) and (9.13), the general base wage and average wage ratios. That is, it is the most general form for two isolated economies.

9.3 Globalization: Capital Mobility and Wages

So far, we have assumed total separation between the two countries. Globalization, however, is about the movement of capital and expertise from one country to another. In all three of our cases, $r_S > r_N$, so capital will flow from the industrial country to the developing country, provided there are no capital controls in either country. In theory, capital will continue to flow to India until $r_S = r_N$. This is what we shall assume.[1]

We shall continue to assume that the parameters in Case 3 hold. Namely, that $A_N = 2.0A_S$ and that $h_N = 2.5h_S$. We do *not* assume, however, that $z_y = 14$. The new value of z_y is determined endogenously when capital is free to move.

First, we see what $r_S = r_N$ implies about relative capital-labor ratios. Use (9.7) to find:

$$\frac{k_N}{k_S} = \frac{h_N}{h_S}\left(\frac{A_N}{A_S}\right)^{\frac{1}{1-\alpha}} \tag{9.25}$$

Using the parameters from Case 3 above — which we consider our most realistic set of parameters — this yields:

$$\frac{k_N}{k_S} = 2.5 * \sqrt{2} = 7.07 \tag{9.26}$$

Even with complete capital mobility, there will be more capital per person in the US than in India. This result is due to the assumption of better technology and more schooling in the US. Both of these make capital more productive in the US, so that if there were *equal* amounts of k, US capital would be more productive and earn a higher return. With diminishing returns to k, in equilibrium more capital remains in the US. Still it is a much

[1] We must remember, however, that the failure of capital to move in sufficient amounts remains a mystery.

smaller gap than in our isolation cases in Section 2.

What does this imply about z_y, relative incomes in the two places? Just use (9.25) in the right-hand side of (9.6) to get:

$$z_y = \frac{h_N}{h_S}\left(\frac{A_N}{A_S}\right)^{\frac{1}{1-\alpha}} = \frac{k_N}{k_S} \tag{9.27}$$

This result is quite interesting. The ratio of real output per worker is the same as that of capital stocks. This means that mobility of capital brings the disparity in income down from 14 to 7.07. It is effectively cut in half.

Notice that if Case 1 were applicable (that is, $A_N = A_S = h_N = h_S = 1$) then complete capital mobility would result in $k_N = k_S$ – evident from (9.25) – which makes real incomes equal by (9.27).

What does capital mobility imply about real base wages in the two countries? Equations (9.12) and (9.13) are still valid. Using the former, and our new value of z_y, and the relative h value from Case 3, the base-wage ratio is:

$$\frac{w_N}{w_S} = z_y \frac{h_S}{h_N} = \frac{7.07}{2.5} = 2.83 \tag{9.28}$$

Compare this to (9.21). The base-wage gap is down to about half of its value without capital mobility. Under what conditions would the gap disappear? We see from (9.27) and (9.28) that if technology were the same in the two nations – $A_N = A_S$ – the wage gap would be eliminated. This happens in the presence of capital mobility even though labor cannot move between countries at all.

It is only base wages that would be equalized. The ratio of *average* wages remains at 7.07 according to (9.13) since the US would have higher average human capital per worker. Arguably, however, the base wage ratio is the more important gap.

9.4 Conclusion

We have seen that the base wage gap depends on two main things. First, is the LDC economy open or closed to capital flows? Second, is LDC technology A similar to, or much lower than, that in the industrial nation?

Only when capital is fully mobile and technology identical will the base-wage gap disappear. Add to that the condition that human capital be the same and the actual average wage gap will also disappear.

Chapter 10

Guidelines for the Empirical Project

10.1 Introduction

The purpose of this project for the student to begin to use economic data for research. The student will use the R programming language to analyze a particular economic question. Both R and its companion R Studio are free.

10.2 The Question

Every research project begins with a question. Data must be available to help find the answer. The best way to proceed is to take a question that can be looked at across many different countries. For example, does a large population help or hinder growth? Or, how much did high rates of female education expenditure help countries grow between 1970 and 2010? This is what is known as “cross-section” analysis.

10.3 The Data

There are two main sources of data for growth and real output, the Penn World Table (PWT, v9.0) and the World Bank (WDI). This data is free and can be downloaded easily. You should go to the PWT website at the University of Groningen to explore the data available there (PWT). Also, see the World Bank website (WDI).

There is, however, far more data on the web. For example, the United Nations publishes a lot of free data. For studies of culture, check out the World Values Survey (WVS). There is no reason to limit yourself. Just check with me before using any data that you find on the internet.

10.4 Organization

The project should be organized along the following lines.

1. Introduction. (A paragraph or two). Explain the question and why you think it is important.

2. Descriptive results. Here you should present a few descriptive graphs and tables using R. (About 2 pages, not counting the graphs and tables.)

3. Econometric analysis. Here you present your regression results. (About 2 pages, not counting tables.)

4. Conclusion. (1 page.)

10.5 Excel and R

Statistical software can read data from text files and Excel files. So, the first step in data analysis, after the data is downloaded from the net, is to put it in Excel. Usually, this is not a problem since most internet data is stored in Excel files.

Table 10.1 shows the form the Excel file should be in to be read most easily by R. (There should be a lot more rows in your data, though.) The first column is the country name; the second column is a three-letter code that identifies the country. All of this data is for one year, 2002, so the year column is optional, but useful to remind you of what the year is for

Table 10.1: Data in the Proper Form

country	*isocode*	*year*	*cg*	*yprch*	*openk*
Afghanistan	AFG	2002	20.89437	525.7682	136.7149
Albania	ALB	2002	20.57526	4134.563	65.64388
Algeria	DZA	2002	25.43648	5723.53	66.85087
Antigua and Barbuda	ATG	2002	65.94318	15274.98	126.0218
Argentina	ARG	2002	17.73209	9561.587	19.36427
Armenia	ARM	2002	22.62712	4338.086	83.46104
Australia	AUS	2002	15.59193	27120.89	44.41226
Austria	AUT	2002	15.9632	27346.25	95.30912
Azerbaijan	AZE	2002	36.39618	3851.348	102.3431
Bahamas, The	BHS	2002	27.87347	18377.36	107.9666
Bahrain	BHR	2002	18.44424	18884.86	168.2105
Bangladesh	BGD	2002	13.78397	2049.461	32.07835
Barbados	BRB	2002	13.74614	15449.81	103.0969
Belarus	BLR	2002	27.6814	11770.66	149.5606
Belgium	BEL	2002	19.37966	25106.27	164.5674
Belize	BLZ	2002	38.79317	6831.726	105.7954
Benin	BEN	2002	10.71865	1311.53	55.67712
Bermuda	BMU	2002	16.56303	33958.16	86.55295
Bhutan	BTN	2002	31.80045	846.4426	69.02751
Bolivia	BOL	2002	19.10262	2971.438	48.10411
Bosnia and Herzegovina	BIH	2002	30.19095	3490.883	69.19069
Botswana	BWA	2002	31.19615	7727.325	88.91344
Brazil	BRA	2002	22.68933	6949.792	23.84154

your project. The important thing is that each column should have a label heading and there should be no blank columns (blank rows are OK). Once one has the data in this form, save it as a comma-delimited (.csv) file. This way it can be easily read into R.

I will give you a template for the data in Excel as well as a template for R code to get you started. Look for it on Blackboard or my webpage (McD).

10.6 Conclusion

The purpose of the empirical project is to get the student involved in basic economic research. By formulating the question, obtaining the data, and analyzing the data with basic graphical and regression methods, the student will get a good idea about the value of the scientific method. It is one thing to propose an idea or theory, and another to find support for it in the data generated by the world.